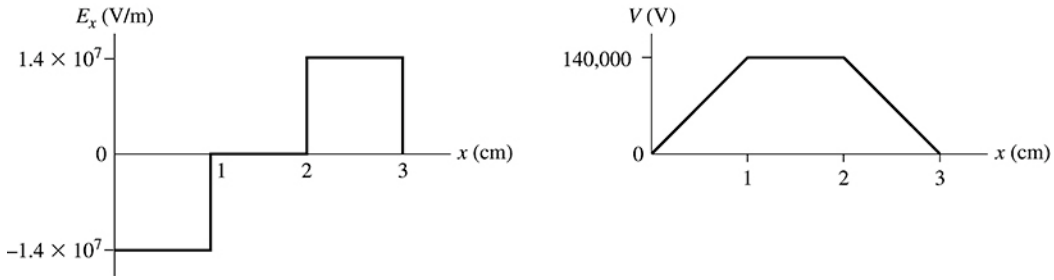
30.41. Model: Assume the electrodes form parallel-plate capacitors with a uniform electric field between the plates.

Visualize:



Please refer to Figure P30.41. The three metal electrodes serve as plates for two capacitors. On the middle electrode, half the charge is located on the left face and half on the right face, thus forming two capacitors. Each plate of the two capacitors carries a charge of ±50 nC.

Solve: **(a)** In the space 0 cm < x < 1 cm, the electric field points to the left and its magnitude is

$$E = \frac{\eta}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{50 \times 10^{-9}\ \text{C}}{(0.02\ \text{m})^2 (8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)} = 1.41 \times 10^7\ \text{V/m}$$

In the region 1 cm ≤ x ≤ 2 cm, $\vec{E} = 0$ because in electrostatics the inside of a conductor has no free charge. The electric field in the region 2 cm < x < 3 cm points to the right and has the same magnitude as the electric field in the region 0 cm < x < 1 cm.

(b) The potential difference between two points in space with a uniform electric field is

$$\Delta V = V_f - V_i = E(x_f - x_i)$$

Assuming that the negative plate at x = 0 m is at zero potential (V_i = 0 V at x_i = 0 cm), $V_f = x_f E$, or simply $V = xE$. Thus, the potential increases linearly with distance x from the negative plate in the region $0 \le x \le 1$. At x = 1 cm, the potential is

$$V = xE = (1.0 \times 10^{-2}\ \text{m})(1.41 \times 10^7\ \text{V/m}) = 1.41 \times 10^{-5}\ \text{V}$$

The potential must be the same throughout the region 1 cm ≤ x ≤ 2 cm. If this were not the case, we would not have an electrostatic situation with the electric field E = 0 V/m. Using the previous reasoning, the potential decreases linearly in the region 2 cm < x < 3 cm.

30.44. Model: Assume the charged rod is a line of charge of length L.

Visualize: Please refer to Figure P30.44.

Solve: **(a)** Divide the charged rod into N small segments, each of length Δx and with charge Δq. The segment i located at position x_i contributes a small amount of potential V_i at point P:

$$V_i = \frac{\Delta q}{4\pi\varepsilon_0 r_i} = \frac{\Delta q}{4\pi\varepsilon_0 (x_0 - x_i)} = \frac{(Q\Delta x/L)}{4\pi\varepsilon_0 (x_0 - x_i)}$$

Point P is at a distance x_0 from the origin. This is done to avoid confusion with x_i. The V_i are now summed and the sum is converted to an integral giving

$$V = \frac{Q}{4\pi\varepsilon_0 L}\int_{-L/2}^{L/2} \frac{dx}{x_0 - x} = \frac{Q}{4\pi\varepsilon_0 L}\left[-\ln(x_0 - x)\right]_{-L/2}^{L/2} = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x_0 + L/2}{x_0 - L/2}\right)$$

Replacing x_0 with x, the potential due to a line charge of length L at a distance x from the center is

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x + L/2}{x - L/2}\right)$$

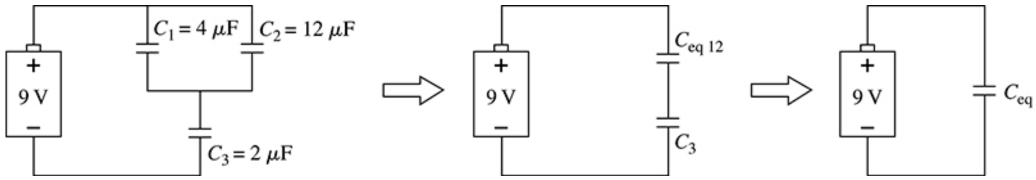
(b) Because $E_x = -dV/dx$,

$$E_x = -\frac{Q}{4\pi\varepsilon_0 L}\frac{d}{dx}\left[\ln(x + L/2) - \ln(x - L/2)\right] = -\frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{(x + L/2)} - \frac{1}{(x - L/2)}\right] = \frac{Q}{4\pi\varepsilon_0}\frac{1}{x^2 - L^2/4}$$

Assess: When $L = 0$ m, $E_x = Q/4\pi\varepsilon_0 x^2$. This is the electric field of a point charge Q a distance x away from a point charge, as expected.

30.61. Model: Assume that the battery is ideal.

Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.

Solve: Because C_1 and C_2 are in parallel,

$$C_{eq\ 12} = C_1 + C_2 = 4\ \mu\text{F} + 12\ \mu\text{F} = 16\ \mu\text{F}$$

$C_{eq\ 12}$ and C_3 are in series, so

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq\ 12}} + \frac{1}{C_3} = \frac{1}{16\ \mu\text{F}} + \frac{1}{2\ \mu\text{F}} = \frac{18}{32}(\mu\text{F})^{-1} \Rightarrow C_{eq} = \tfrac{32}{18}\ \mu\text{F}$$

A potential difference of $\Delta V_C = 9$ V across a capacitor of equivalent capacitance $\tfrac{32}{18}\ \mu$F produces a charge

$$Q = C_{eq}\Delta V_C = \left(\tfrac{32}{18}\ \mu\text{F}\right)9\text{ V} = 16\ \mu\text{C}$$

Because C_{eq} is a series combination of two capacitors $C_{eq\ 12}$ and C_3, $Q_3 = Q_{eq\ 12} = 16\ \mu$C. The potential difference across C_3 is

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{16\ \mu\text{C}}{2\ \mu\text{F}} = 8.0\text{ V}$$

Now, $Q_{eq\ 12} = 16\ \mu$C is the charge on the equivalent capacitor with $C_{eq\ 12} = 16\ \mu$F. So, the potential difference across the equivalent capacitor $C_{eq\ 12}$ is

$$\Delta V_{eq\ 12} = \frac{Q_{eq\ 12}}{C_{eq\ 12}} = \frac{16\ \mu\text{C}}{16\ \mu\text{F}} = 1.0\text{ V}$$

Parallel capacitors C_1 and C_2 have the same potential difference as the equivalent capacitor $C_{eq\ 12}$, so $\Delta V_1 = \Delta V_2 =$ 1.0 V. The charge on each is given by $Q = C\Delta V$, so $Q_1 = (4\ \mu\text{F})(1.0\text{ V}) = 4.0\ \mu$C and $Q_2 = (12\ \mu\text{F})(1.0\text{ V}) = 12.0\ \mu$C.

In summary, $Q_1 = 4.0\ \mu$C, $\Delta V_1 = 1.0$ V; $Q_2 = 12.0\ \mu$C, $\Delta V_2 = 1.0$ V; and $Q_3 = 16.0\ \mu$C, $\Delta V_3 = 8.0$ V.

Assess: Note that $\Delta V_3 + \Delta V_{eq\ 12} = 9.0\text{ V} = \Delta V_{bat}$, as it should. Also that $Q_1 + Q_2 = 16.0\ \mu\text{C} = Q_{eq\ 12}$, as it should.

30.67. Model: Assume the battery is ideal.

Visualize: Please refer to Figure P30.67. While the switch is in position A, the capacitors C_2 and C_3 are uncharged. When the switch is placed in position B, the charged capacitor C_1 is connected to C_2 and C_3. C_2 and C_3 are connected in series to form an equivalent capacitor $C_{\text{eq 23}}$.

Solve: While the switch is in position A, a potential difference of $V_1 = 100$ V across C_1 charges it to

$$Q_1 = C_1V_1 = \left(15\times10^{-6}\text{ F}\right)\left(100\text{ V}\right) = 1500\ \mu\text{C}$$

When the switch is moved to position B, this initial charge Q_1 is redistributed. The charge Q_1' goes on C_1 and the charge $Q_{\text{eq 23}}$ goes on $C_{\text{eq 23}}$. The voltage across C_1 and $C_{\text{eq 23}}$ is the same and $Q_1' + Q_{\text{eq 23}} = Q_1 = 1500\ \mu\text{C}$. Combining these two conditions, we get

$$\frac{Q_1'}{C_1} = \frac{Q_{\text{eq 23}}}{C_{\text{eq 23}}} \Rightarrow \frac{1500\ \mu\text{C} - Q_{\text{eq 23}}}{C_1} = \frac{Q_{\text{eq 23}}}{C_{\text{eq 23}}}$$

Since $C_{\text{eq 23}} = \left(\frac{1}{20\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}}\right)^{-1} = 12\ \mu\text{F},$ we can rewrite this equation as

$$\frac{1500\ \mu\text{C} - Q_{\text{eq 23}}}{15\ \mu\text{F}} = \frac{Q_{\text{eq 23}}}{12\ \mu\text{F}} \Rightarrow Q_{\text{eq 23}} = 0.67\text{ mC} \Rightarrow Q_1' = Q_1 - Q_{\text{eq 23}} = 1.500\text{ mC} - 0.67\text{ mC} = 0.83\text{ mC}$$

Having found the charge $Q_{\text{eq 23}}$, it is easy to see that $Q_2 = Q_3 = 0.67$ mC because $C_{\text{eq 23}}$ is a series combination of C_2 and C_3. Thus,

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{670\ \mu\text{C}}{20\ \mu\text{F}} = 34\text{ V} \qquad \Delta V_3 = \frac{Q_3}{C_3} = \frac{670\ \mu\text{C}}{30\ \mu\text{F}} = 22\text{ V} \qquad \Delta V_1 = \frac{Q_1'}{C_1} = \frac{830\ \mu\text{C}}{15\ \mu\text{F}} = 55\text{ V}$$

30.71. Solve: (a) The capacitance of the parallel-plate capacitor is

$$C_1 = \frac{A\varepsilon_0}{d_1} = \frac{(0.10 \text{ m} \times 0.10 \text{ m})(8.85\times10^{-12} \text{ C}^2/\text{N m}^2)}{1.0\times10^{-3} \text{ m}} = 8.85\times10^{-11} \text{ F}$$

The electric potential energy stored in the capacitor is

$$U_1 = \frac{1}{2}\frac{Q^2}{C_1} = \frac{1}{2}\frac{(10\times10^{-9} \text{ C})^2}{8.85\times10^{-11}\text{F}} = 5.7\times10^{-7} \text{ J}$$

(b) There is no change in the charge. The energy change is due to the change in the capacitance. The new capacitance is

$$C_2 = \frac{\varepsilon_0 A}{d_2} = \frac{\varepsilon_0 A}{2d_1} = \frac{C_1}{2}$$

The amount of energy stored is $U_2 = 11.4\times10^{-7}$ J.

(c) Work was done on the capacitor by the agent pulling the plates apart, thereby adding energy into the system.