WORKSHOP 6.1 2/18/2009

29-42

Energy is conserved

electric potential at point A

$$E_A = \frac{1}{2} m_{p+} v_A^2 + q_{p+} V_A$$

initial velocity

$$E_B = \frac{1}{2} m_{p+} v_B^2 + q_{p+} V_B$$

$$E_A = E_B$$

$$\Rightarrow \frac{1}{2} m_{p+} v_A^2 + q_{p+} V_A = \frac{1}{2} m_{p+} v_B^2 + q_{p+} V_B$$

$$v_B = \left[v_A^2 + \frac{2 q_{p+} V_A}{m_{p+}} - \frac{2 q_{p+} V_B}{m_{p+}} \right]^{1/2}$$

$$= \left[(50{,}000\,\text{m/s})^2 + \frac{2(1.6\times10^{-19}\,\text{C})}{(1.67\times10^{-27}\,\text{kg})} (30\,\text{V} - (-10\,\text{V})) \right]^{1/2}$$

$$\boxed{v_B = 1.01\times10^5\,\text{m/s}}$$

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a) The charge moves rapidly to the outside surface to ensure $\vec{E}_{conductor} = 0$

b) $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$ (same as for point charge)

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r} \Rightarrow \boxed{q = 4\pi\varepsilon_0 r V}$$

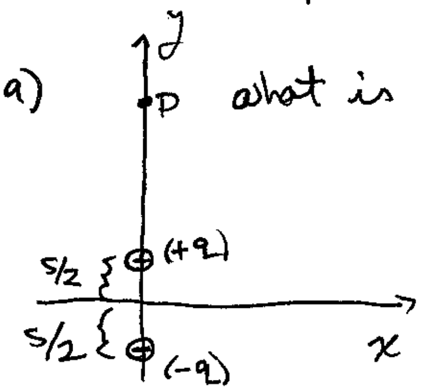
$$= 4\pi\varepsilon_0 \left(\frac{.30\,\text{m}}{2}\right)(500{,}000\,\text{V})$$

$$\boxed{q = 8.3\times10^{-6}\,\text{C}}$$

c) $\boxed{\vec{E}_{inside} = 0}$ since $\vec{E}$ is always zero inside a conductor.

$$\boxed{\vec{E}_{outside} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} = \frac{(8.3\times10^{-6}\,\text{C})}{4\pi\varepsilon_0 (.15\,\text{m})^2} \hat{r} = 3.3\times10^6\,\text{V/m}}$$

29-66 Dipole oriented along y-axis



a) what is V at point P? $P=(0,y)$

$$V_{tot} = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(y-s/2)} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(y+s/2)}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{y-s/2} - \frac{1}{y+s/2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{s}{y^2-s^2/4}\right]$$

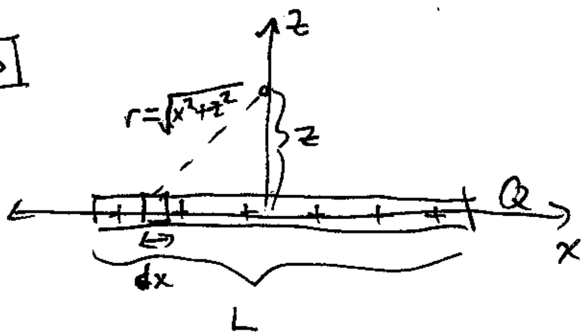
$$= \frac{|\vec{p}|}{4\pi\varepsilon_0 (y^2 - s^2/4)} \overset{y \gg s}{\approx} \frac{|\vec{p}|}{4\pi\varepsilon_0 y^2}$$

$$\boxed{V_{tot} \approx \frac{p}{4\pi\varepsilon_0 y^2}}$$

b) Water molecule has $p = 6.2\times10^{-30}\,\mathrm{C\cdot m}$

$$\boxed{V_{tot} = \frac{6.2\times10^{-30}\,\mathrm{C\cdot m}}{4\pi\left(8.85\times10^{-12}\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)(1\times10^{-9}\,\mathrm{m})^2} = .056\,\mathrm{V}}$$

29-70



rod has linear charge density $\lambda = \frac{Q}{L}$

$$V_{tot} = \int \frac{dQ}{4\pi\varepsilon_0 r} \qquad dQ = \lambda\,dx = \frac{Q}{L}dx$$

$$r = \sqrt{x^2+z^2}$$

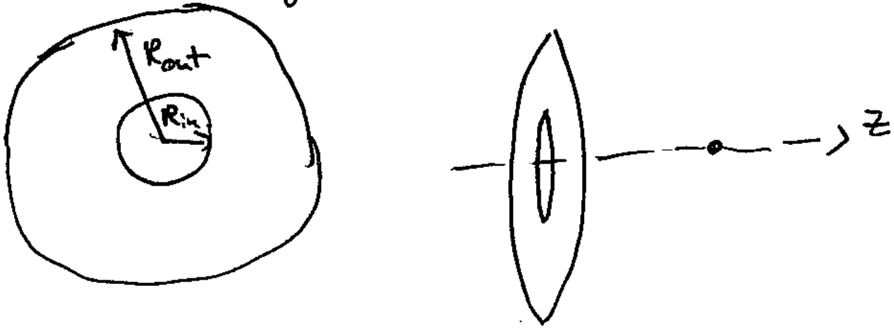
$$V_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\int_{-L/2}^{L/2} dx\,\frac{1}{\sqrt{x^2+z^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\,2\log\left(\frac{L+\sqrt{L^2+4z^2}}{2z}\right)$$

$$\boxed{V_{tot} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{L}\log\left(\frac{L+\sqrt{L^2+4z^2}}{2z}\right)}$$

$\boxed{29-72}$ Disc with hole

charge Q



Example 29.12 has set up most of the problem for you

new charge density $\eta = \dfrac{Q}{A} = \dfrac{Q}{\pi(R_{out}^2 - R_{in}^2)}$

for a ring with charge dQ, have $V_{ring} = \dfrac{dQ}{4\pi\varepsilon_0\sqrt{r^2+z^2}}$

and $dQ = \eta\, dA = \dfrac{Q}{\pi(R_{out}^2 - R_{in}^2)} \cdot 2\pi r\, dr$

$$V_{tot} = \int_{R_{in}}^{R_{out}} dr \frac{dQ}{4\pi\varepsilon_0\sqrt{r^2+z^2}} = \frac{Q}{2\pi\varepsilon_0(R_{out}^2 - R_{in}^2)} \int_{R_{in}}^{R_{out}} dr \frac{r}{\sqrt{r^2+z^2}}$$

$$\boxed{V_{tot} = \frac{Q}{2\pi\varepsilon_0(R_{out}^2 - R_{in}^2)}\left[\sqrt{R_{out}^2 - z^2} - \sqrt{R_{in}^2 + z^2}\right]}$$

for $R_{in} = 0$, we have $\sqrt{R_{in}^2 + z^2} \rightarrow \sqrt{z^2} = |z|$

$$\Rightarrow \boxed{V_{tot}(R_{in}=0) = \frac{Q}{2\pi\varepsilon_0(R_{out}^2)}\left[\sqrt{R_{out}^2 - z^2} - |z|\right]}$$

which agrees with the result in Ex. 29.12