WORKSHOP 5.1

28-35 $\vec{E} = (5000\, r^2)\hat{r}$ N/C

a) at 20cm (or .2m) $\vec{E} = \left(5000 \frac{(.2m)^2}{m^2}\right)\hat{r}$ N/C

$$\boxed{\vec{E}_{20cm} = 200\hat{r} \text{ N/C}}$$

b) $\Phi_E = \oint \vec{E}\cdot d\vec{A}$ ~~so~~ at a surface with ~~diameter~~ 40 cm (radius 20cm)

since $\vec{E}$ is $\parallel$ with $d\vec{A}$

$$\oint_{20cm} \vec{E}\cdot dA = |E| \oint_{20cm} dA = 4\pi r^2 |E| = 4\pi (.2m)^2 (200\text{N/C})$$

$$\boxed{\Phi_E = 100.5 \frac{\text{N m}^2}{\text{C}}}$$

c) $\Phi_E = \frac{Q_{enc}}{\varepsilon_0}$ so $Q_{enc} = \Phi_E \varepsilon_0 = \left(100.5 \frac{\text{Nm}^2}{\text{C}}\right)\left(8.85\times10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)$

$$\boxed{Q_{enc} = 8.9\times10^{-10}\text{C} = .89\text{nC}}$$

28-40

a) Gauss' law tells us that ~~for~~ outside a sperical charge distribution,

$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{r^2}\hat{r}$ here, at 8cm, $\vec{E} = +\frac{1}{4\pi\varepsilon_0}\frac{Q}{(.08m)^2} = -\hat{r}\,15{,}000$ N/C

$$\boxed{Q = -1.07\times10^{-8}\text{C}}$$

b) $\vec{E}_{\text{inside conductor}} = 0$ and thus for $10cm < r < 15cm$, $Q_{enc} = 0$

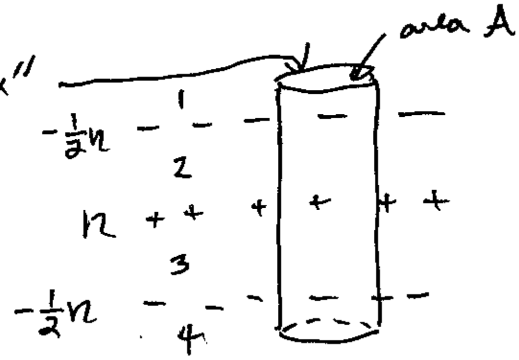
by Gauss' Law and thus $\boxed{Q_{\text{inside surface of hollow sphere}} = +1.07\times10^{-8}\text{C}}$

c) now at 17cm $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{(.17m)^2} = +\hat{r}\,15{,}000$ N/C

and $\boxed{Q_{enc} = 4.82\times10^{-8}\text{C}}$ → this is the charge on the exterior surface since $Q_{a)} + Q_{b)} = 0$ and $Q_{enc} = Q_{a)} + Q_{b)} + Q_{c)}$

28-47

Regions 1 and 4: Gaussian "pill-box"

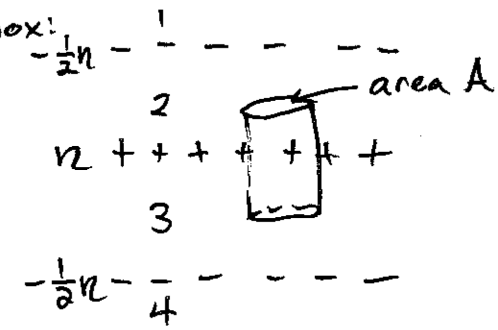


$$Q_{enc} = A\left(-\tfrac{1}{2}n + n - \tfrac{1}{2}n\right) = 0$$

and by symmetry $\Phi_{top} = \Phi_{bottom} = \dfrac{Q_{enc}}{2\varepsilon_0} = 0$

thus $\boxed{\vec{E}_1 = \vec{E}_4 = 0}$

Regions 2 and 3: New Gaussian Pill-box:



$\dfrac{Q_{enc}}{\varepsilon_0} = \dfrac{nA}{\varepsilon_0} = \Phi_{top} + \Phi_{bottom}$ and again, by reflection symmetry, $\Phi_{top} = \Phi_{bottom}$

$\vec{E} \parallel$ to dA

$$\Phi_{top} = \frac{nA}{2\varepsilon_0} = \int_{top} \vec{E}\cdot d\vec{A} = |E|\int_{top} dA = |E|A \Rightarrow \vec{E}_2 = \left(\frac{n}{2\varepsilon_0}, \text{up}\right)$$

since $d\vec{A}$ points up

and $\vec{E}_3 = \left(\dfrac{n}{2\varepsilon_0}, \text{down}\right)$

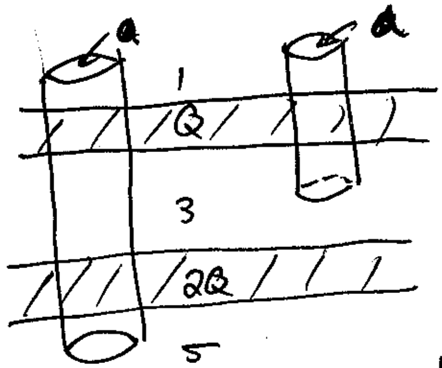
↳ since now $d\vec{A}$ points <u>down</u>

$\boxed{28-50}$ a) Regions 2 and 4 are inside conductors, and thus $\boxed{\vec{E}_2 = \vec{E}_4 = 0}$

for 1, 3, and 5

first, note $\vec{E}_1 = -\vec{E}_5$. This is due to the fact that sheets of charge generate E-fields that are independent of distance from the sheet.

Gaussian surfaces



since $\vec{E}_1 = -\vec{E}_5$, Gauss' law using the left cylinder gives

$$\frac{Q_{enc}}{\varepsilon_0} = \frac{3Q\frac{a}{A}}{\varepsilon_0} = |E_1|a + |E_5|a$$

and $|E_1| = \frac{3}{2}\frac{Q/A}{\varepsilon_0}$ $\quad |E_5| = \frac{3}{2}\frac{Q/A}{\varepsilon_0}$

$$\boxed{\vec{E}_1 = \left(\frac{3}{2}\frac{Q/A}{\varepsilon_0}, \text{up}\right) \quad \vec{E}_5 = \left(\frac{3}{2}\frac{Q/A}{\varepsilon_0}, \text{down}\right)}$$

now use the right cylinder: $\Phi_E = |\vec{E}_1|a \pm |\vec{E}_3|a = \frac{Q_{enc}}{\varepsilon_0} = \frac{Q/A \cdot a}{\varepsilon_0}$

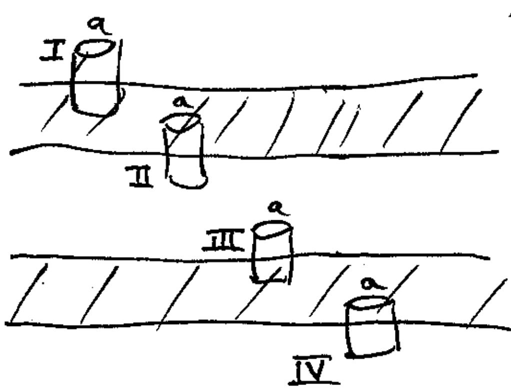
$\hookrightarrow$ + for $\vec{E}_3$ down, − for up

thus $\frac{3}{2}\frac{Q/A}{\varepsilon_0} \pm |\vec{E}_3| = \frac{Q/A}{\varepsilon_0}$ and thus

$\pm|\vec{E}_3| = -\frac{Q/A}{2\varepsilon_0}$ — must be correct, so

$$\boxed{\vec{E}_3 = \left(\frac{Q/A}{2\varepsilon_0}, \text{up}\right)}$$

b) Now use these Gaussian surfaces:



I. $\Phi = |\vec{E}_1|a = \frac{\eta_a a}{\varepsilon_0} \Rightarrow \eta_a = \frac{3}{2}\,Q/A$

II. $\Phi = -|\vec{E}_3|a = \frac{\eta_b a}{\varepsilon_0} \Rightarrow \eta_b = -\frac{1}{2}\,Q/A$

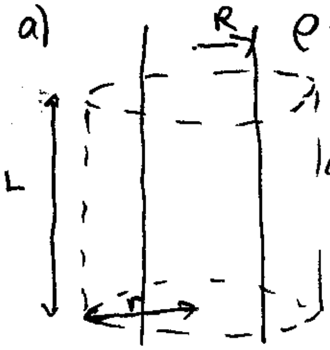
III. $\Phi = +|\vec{E}_3|a = \frac{\eta_c a}{\varepsilon_0} \Rightarrow \eta_c = +\frac{1}{2}\,Q/A$

IV. $\Phi = |\vec{E}_5|a = \frac{\eta_d a}{\varepsilon_0} \Rightarrow \eta_d = +\frac{3}{2}\,Q/A$

28-52

$r \geq R$:

a)



$\rho = \rho_0$ (constant charge density) $\qquad \lambda = \rho_0 \cdot \pi R^2 \Rightarrow \rho_0 = \frac{\lambda}{\pi R^2}$

← Gaussian surface $\qquad \Phi_{top} = \Phi_{bottom} = 0$ since $\vec{E}$ must be $\perp$ to $d\vec{A}$

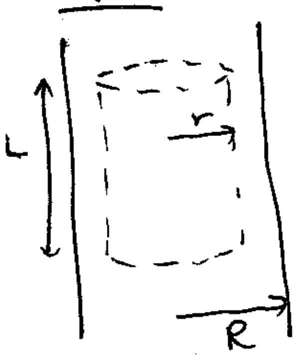
on outside, by symmetry, $\vec{E}$ is $\parallel$ to $d\vec{A}$

$$\Phi_{out} = \oint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \hat{r} \oint dA = \vec{E} \cdot \hat{r} \; 2\pi r L = \frac{Q_{enc}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$\vec{E} \cdot \hat{r} = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow \boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r} \hat{r}}$$

Usual formula for line of charge!

b) $r < R$:



now $Q_{enc} = \rho_0 V = \frac{\lambda}{\pi R^2} \cdot \pi r^2 L = \lambda L \frac{r^2}{R^2}$ ← V: volume of Gaussian surface

$$\Phi_{out} = \oint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \hat{r} \oint_{out} dA = \vec{E} \cdot \hat{r} \, (2\pi r L) = \frac{Q_{enc}}{\varepsilon_0} = \frac{\lambda L r^2}{\varepsilon_0 R^2}$$

$$\Rightarrow \vec{E} \cdot \hat{r} = \frac{\lambda r}{2\pi\varepsilon_0 R^2} \Rightarrow \boxed{\vec{E} = \frac{1}{2\pi\varepsilon_0} \frac{\lambda r}{R^2} \hat{r}}$$