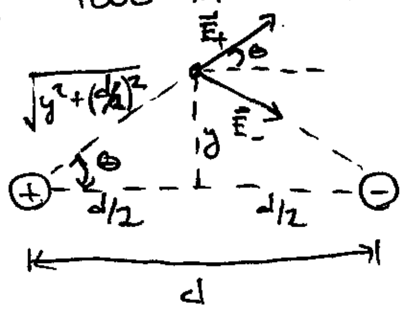
Workshop 3.1

27.37 Two infinite lines of charge (perpendicular to page)

a)



note that $\hat{j}$ component vanishes!

also $(\vec{E}_+)_x = (\vec{E}_-)_x = |\vec{E}_+| \cos\theta$

$\rightarrow$ So $\vec{E}_{tot} = 2|\vec{E}_+|\cos\theta\, \hat{i}$

$\left(\vec{E}_{tot} = [(\vec{E}_+)_x + (\vec{E}_-)_x]\,\hat{i}\right)$

$$|\vec{E}_+| = K\left(\frac{2\lambda}{r}\right) = \frac{2K\lambda}{\sqrt{y+(d/2)^2}} \qquad \cos\theta = \frac{d/2}{\sqrt{y^2+(d/2)^2}}$$

$$\Rightarrow \boxed{\vec{E}_{tot} = \frac{2K\lambda d}{y^2+(d/2)^2}\,\hat{i} = \frac{1}{2\pi\varepsilon_0}\,\frac{\lambda d}{y^2+(d/2)^2}\,\hat{i}}$$

b) We only want some key features of the plot:

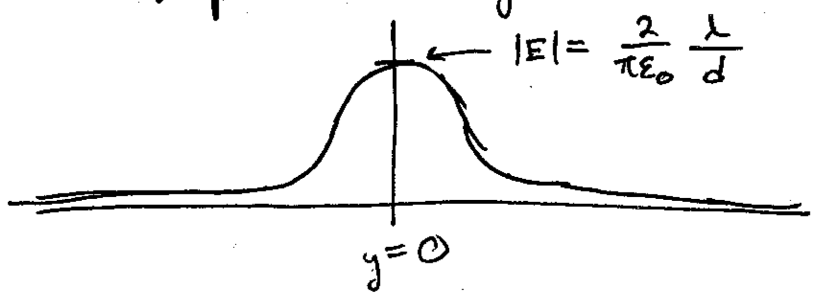
i) symmetric about $y=0$

ii) derivative with respect to y is smooth everywhere (no "kinks" in the plot)

iii) goes to zero for large y

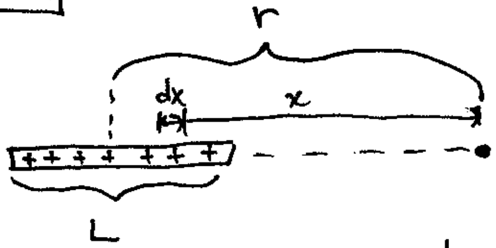
iv) is not vanishing at $y=0$

v) positive everywhere for positive λ



$\boxed{27.42}$ Charged rod with charge Q

a)



$\vec{E}$ is in the $\hat{i}$ direction:

$$\vec{E}_{tot} = \frac{\hat{i}}{4\pi\varepsilon_0}\int \frac{dQ}{x^2}$$

charge density

$$dQ = \lambda dx = \frac{Q}{L}dx \Rightarrow \vec{E}_{tot} = \frac{\hat{i}}{4\pi\varepsilon_0}\int_{r-L/2}^{r+L/2}\frac{Q}{L}\frac{dx}{x^2}$$

$$= \frac{\hat{i}}{4\pi\varepsilon_0}\frac{Q}{L}\left(-\frac{1}{x}\right)\Bigg|_{r-L/2}^{r+L/2}$$

$$= \frac{\hat{i}}{4\pi\varepsilon_0}\frac{Q}{L}\left[\frac{1}{r-L/2} - \frac{1}{r+L/2}\right]$$

$$\boxed{\vec{E}_{tot} = \frac{\hat{i}}{4\pi\varepsilon_0}\frac{Q}{r^2-(L/2)^2}}$$

b) for $r \gg L/2$, this formula is approximately

$$\boxed{\vec{E}_{tot} \overset{r \gg L/2}{\simeq} \frac{\hat{i}}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

the same formula as for a point charge!

★ All charge distributions with nonvanishing total charge Q behave like point charges if you get far enough away!

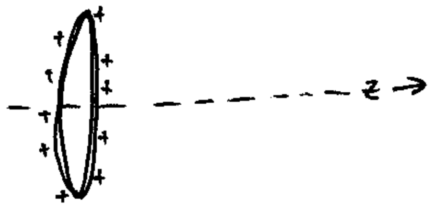
c) $L = .05\,m$

$r = .03\,m$

$Q = 3\,nC$

$$\boxed{\vec{E}_{tot} = \frac{(9\times10^9\,Nm^2/C^2)(3\times10^{-9}\,C)}{(.03m)^2 - \left(\frac{.05m}{2}\right)^2}\hat{i} = 9.8\times10^4\,N/C\,\hat{i}}$$

$\boxed{27.44}$ For a ring of charge, on axis, the $\vec{E}$ field is given by



$$(\vec{E}_{ring})_z = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$

for $z \ll R$ re-organize formula:

$$(\vec{E}_{ring})_z = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{R^3}\frac{1}{(1+(z/R)^2)^{3/2}}$$

Taylor expand in $\left(\frac{z}{R}\right)$: terms of order $(z/R)^3$ or higher

$$(\vec{E}_{ring})_z = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\Big[\underbrace{\left(\frac{z}{R}\right)}_{\text{leading term}} + \underbrace{\mathcal{O}\left(\left(\frac{z}{R}\right)^3\right)}_{\text{subleading terms}}\Big]$$

$$\boxed{(\vec{E}_{ring})_z \simeq 0}$$

for $R \ll z$ re-organize a different way

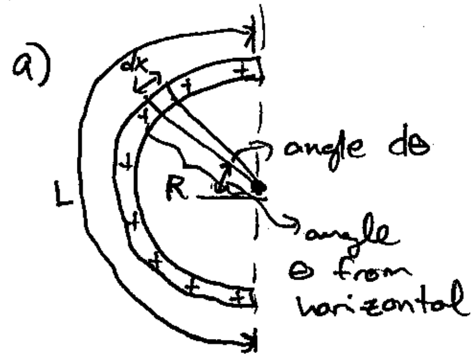
$$(\vec{E}_{ring})_z = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{z^3}\frac{1}{[1+(R/z)^2]^{3/2}} \overset{\text{Taylor expand in } (R/z)}{=} \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}\left[1+\mathcal{O}\left(\left(\frac{R}{z}\right)^2\right)\right]$$

leading term is $\boxed{(\vec{E}_{ring})_z \simeq \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}}$

Again, we see Coulomb's Law emerge at a large distance from a complicated charge distribution!
(see last problem)

$\boxed{27.46}$

a)



We want the E field at the center of curvature for a half ring charge distribution

Note: by symmetry, $\hat{j}$ component of $\vec{E}$ is zero!

$$\vec{E}_{tot} = \frac{\hat{i}}{4\pi\varepsilon_0}\int \frac{dQ}{R^2}\cos\theta$$

$\uparrow$ to get x-component of $d\vec{E}$ due to charge dQ

charge density λ, $dQ = \lambda\, dx$

$\lambda = \frac{Q}{L}$ $\qquad$ $dx = R\,d\theta = \frac{L}{\pi}d\theta$ $\qquad \Rightarrow \boxed{dQ = \frac{Q}{\pi}d\theta}$

$\uparrow$ in radians!

$$\Rightarrow \vec{E}_{tot} = \frac{\hat{i}}{4\pi\varepsilon_0}\frac{1}{(L/\pi)^2}\int_{-\pi/2}^{\pi/2}\frac{Q}{\pi}\cos\theta\, d\theta = \frac{\hat{i}}{4\pi\varepsilon_0}\frac{Q\pi}{L^2}\underbrace{(\sin\theta)\Big|_{-\pi/2}^{\pi/2}}_{2}$$

$$\boxed{\vec{E}_{tot} = \frac{\hat{i}}{2\varepsilon_0 L^2}Q}$$

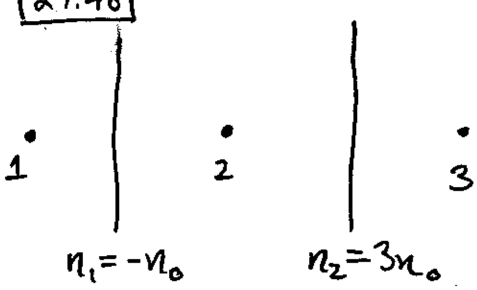
b) $Q = 30\,nC$

$L = .10m$

$$\boxed{\vec{E}_{tot} = \frac{2\pi(9\times10^9\,Nm^2/C^2)(30\times10^{-9}C)}{(.10m)^2} = 1.70\times10^5\,N/C}$$

$\boxed{27.48}$

Find E-field at points 1, 2, & 3



$\vec{E}_{sheet} = \frac{\eta}{2\varepsilon_0}$ away from sheet

$$\boxed{\vec{E}_1 = (-\hat{i})\left[\frac{\eta_1}{2\varepsilon_0}+\frac{\eta_2}{2\varepsilon_0}\right] = (-\hat{i})\left[\frac{-\eta_0}{2\varepsilon_0}+\frac{3\eta_0}{2\varepsilon_0}\right] = -\frac{\eta_0}{\varepsilon_0}\hat{i}}$$

$$\boxed{\vec{E}_2 = \frac{\eta_1}{2\varepsilon_0}\hat{i} + \frac{\eta_2}{2\varepsilon_0}(-\hat{i}) = -\frac{2\eta_0}{\varepsilon_0}\hat{i}}$$

$$\boxed{\vec{E}_3 = (+\hat{i})\left[\frac{\eta_1}{2\varepsilon_0}+\frac{\eta_2}{2\varepsilon_0}\right] = \frac{\eta_0}{\varepsilon_0}\hat{i}}$$