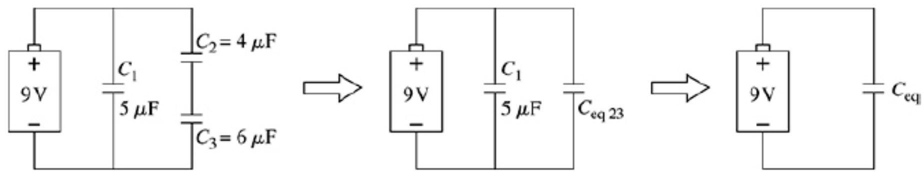
30.62. Model: Assume the battery is an ideal battery.
Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.
Solve: Because C_2 and C_3 are in series,

$$\frac{1}{C_{eq\,23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{4\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}} = \frac{10}{24}(\mu\text{F})^{-1} \Rightarrow C_{eq\,23} = \tfrac{24}{10}\mu\text{F} = 2.4\ \mu\text{F}$$

$C_{eq\,23}$ and C_1 are in parallel, so

$$C_{eq} = C_{eq\,23} + C_1 = 2.4\ \mu\text{F} + 5\ \mu\text{F} = 7.4\ \mu\text{F}$$

A potential difference of $\Delta V_C = 9$ V across a capacitor of equivalent capacitance 7.4 μF produces a charge

$$Q = C_{eq}\Delta V_C = (7.4\ \mu\text{F})(9\ \text{V}) = 66.6\ \mu\text{C}$$

Because C_{eq} is a parallel combination of C_1 and $C_{eq\,23}$, these capacitors have $\Delta V_1 = \Delta V_{eq\,23} = \Delta V_C = 9$ V. Thus the charges on these two capacitors are

$$Q_1 = (5\ \mu\text{F})(9\ \text{V}) = 45\ \mu\text{C} \qquad Q_{eq\,23} = (2.4\ \mu\text{F})(9\ \text{V}) = 21.6\ \mu\text{C}$$

Because $Q_{eq\,23}$ is due to a series combination of C_2 and C_3, $Q_2 = Q_3 = 21.6\ \mu$C. This means

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{21.6\ \mu\text{C}}{4\ \mu\text{F}} = 5.4\ \text{V} \qquad \Delta V_3 = \frac{Q_3}{C_3} = \frac{21.6\ \mu\text{C}}{6\ \mu\text{F}} = 3.6\ \text{V}$$

In summary, $Q_1 = 45\ \mu$C, $V_1 = 9$ V; $Q_2 = 21.6\ \mu$C, $V_2 = 5.4$ V; and $Q_3 = 21.6\ \mu$C, $V_3 = 3.6$ V.

30.68. Model: Assume the battery is an ideal battery.
Visualize: Please refer to Figure P30.68. The battery is connected to two series capacitors C_1 and C_2.
Solve: The equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Because the charge on capacitor C_2 is 450 μC, the charge on C_{eq} and C_1 is also 450 μC. We have

$$\Delta V_{eq} = \frac{Q_{eq}}{C_{eq}} \Rightarrow 60\ \text{V} = \frac{450\ \mu\text{C}}{C_1 C_2/(C_1 + C_2)} = \frac{(450\ \mu\text{C})(12\ \mu\text{F} + C_2)}{(12\ \mu\text{F})C_2} \Rightarrow C_2 = \frac{(450\ \mu\text{C})(12\mu\text{F})}{720\ \mu\text{C} - 450\ \mu\text{C}} = 20\ \mu\text{F}$$

Assess: Note that capacitors connected in series have the same charge.

30.70. Solve: The magnitude of the work done by the external force is equal to the change in the electric potential energy of the capacitor. The work done is

$$W_{\text{force}} = \Delta U = U_2 - U_1 = \frac{1}{2}\frac{Q^2}{C_2} - \frac{1}{2}\frac{Q^2}{C_1}$$

Note that the charge on the plates is not changed as the distance between the electrodes is changed. Thus,

$$W_{\text{force}} = \tfrac{1}{2}Q^2\left[\frac{1}{C_2} - \frac{1}{C_1}\right] = \tfrac{1}{2}\left(4.0\times10^{-3}\ \text{C}\right)^2\left[\frac{1}{2.0\ \mu\text{F}} - \frac{1}{5.0\ \mu\text{F}}\right] = 2.4\ \text{J}$$

Assess: The work done on the capacitor is stored as electric potential energy in the capacitor.

30.72. Solve: The energy density in the electric field at the surface of the sphere is $u = \tfrac{1}{2}\varepsilon_0 E^2$, where E is the electric field strength at the surface. The electric field strength and the potential on the surface of the sphere are related as follows:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\right)\frac{1}{R} = \frac{V}{R} = \frac{1000\ \text{V}}{5.0\times10^{-3}\ \text{m}} = 2.0\times10^5\ \text{N/m}$$

$$\Rightarrow u = \tfrac{1}{2}\left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right)\left(2.0\times10^5\ \text{V/m}\right)^2 = 0.177\ \text{J/m}^3$$