

**29.45. Model:** Energy is conserved. The proton's potential energy inside the capacitor can be found from the capacitor's potential difference.

**Visualize:** Please refer to Figure P29.45.

**Solve:** (a) The electric potential at the midpoint of the capacitor is 250 V. This is because the potential inside a parallel-plate capacitor is  $V = Es$  where  $s$  is the distance from the negative electron. The proton has charge  $q = e$  and its potential energy at a point where the capacitor's potential is  $V$  is  $U = eV$ . The proton will gain potential energy

$$\Delta U = e\Delta V = e(250 \text{ V}) = 1.60 \times 10^{-19} \text{ C} (250 \text{ V}) = 4.00 \times 10^{-17} \text{ J}$$

if it moves all the way to the positive plate. This increase in potential energy comes at the expense of kinetic energy which is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(200,000 \text{ m/s})^2 = 3.34 \times 10^{-17} \text{ J}$$

This available kinetic energy is not enough to provide for the increase in potential energy if the proton is to reach the positive plate. Thus the proton does not reach the plate because  $K < \Delta U$ .

(b) The energy-conservation equation  $K_f + U_f = K_i + U_i$  is

$$\begin{aligned} \frac{1}{2}mv_f^2 + qV_f &= \frac{1}{2}mv_i^2 + qV_i \Rightarrow \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + q(V_i - V_f) \\ \Rightarrow v_f &= \sqrt{v_i^2 + \frac{2q}{m}(V_i - V_f)} = \sqrt{(2.0 \times 10^5 \text{ m/s})^2 + \frac{2(1.60 \times 10^{-19} \text{ C})(250 \text{ V} - 0 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}} = 2.96 \times 10^5 \text{ m/s} \end{aligned}$$

**29.64. Solve:** A charged particle placed inside a uniformly charged spherical shell experiences no electric force. That is,  $E = 0 \text{ V/m}$  inside the shell. We know from Section 29.5 that a difference in potential between two points or two plates is the source of an electric field. Since the potential on the surface of the shell is  $V = Q/4\pi\epsilon_0 R$ , the potential inside must be the same. This ensures that the potential difference is zero and hence the electric field is zero inside the shell. The potential at the center of the spherical shell is thus the same as at the surface. That is,  $V_{\text{center}} = V_{\text{surface}} = Q/4\pi\epsilon_0 R$ .

**29.69. Model:** Assume the thin rod is a line of charge with uniform linear charge density.

**Visualize:** Please refer to Figure P29.69. The point P is a distance  $d$  from the origin. Divide the charged rod into  $N$  small segments, each of length  $\Delta x$  and with charge  $\Delta q$ . Segment  $i$ , located at position  $x_i$ , contributes a small amount of potential  $V_i$  at point P.

**Solve:** The contribution of the  $i$ th segment is

$$V_i = \frac{\Delta q}{4\pi\epsilon_0 r_i} = \frac{\Delta q}{4\pi\epsilon_0 (d - x_i)} = \frac{Q\Delta x/L}{4\pi\epsilon_0 (d - x_i)}$$

where  $\Delta q = \lambda\Delta x$  and the linear charge density is  $\lambda = Q/L$ . We are placing the point P at a distance  $d$  rather than  $x$  from the origin to avoid confusion with  $x_i$ . The  $V_i$  are now summed and the sum is converted to an integral giving

$$V = \frac{Q}{4\pi\epsilon_0 L} \int_{-L/2}^{L/2} \frac{dx}{d - x} = \frac{Q}{4\pi\epsilon_0 L} [-\ln(d - x)]_{-L/2}^{L/2} = \frac{Q}{4\pi\epsilon_0 L} \ln\left(\frac{d + L/2}{d - L/2}\right)$$

Replacing  $d$  with  $x$ , the potential due to a line charge of length  $L$  at a distance  $x$  along the axis is

$$V = \frac{Q}{4\pi\epsilon_0 L} \ln\left(\frac{x + L/2}{x - L/2}\right)$$

**29.71. Model:** Because the rod is thin, assume the charge lies along the semicircle of radius  $R$ .

**Visualize:** Please refer to Figure P29.71. The bent rod lies in the  $xy$ -plane with point P as the center of the semicircle. Divide the semicircle into  $N$  small segments of length  $\Delta s$  and of charge  $\Delta Q = (Q/\pi R)\Delta s$ , each of which can be modeled as a point charge. The potential  $V$  at P is the sum of the potentials due to each segment of charge.

**Solve:** The total potential is

$$V = \sum_i V_i = \sum_i \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{R} = \frac{1}{4\pi\epsilon_0 R} \sum_i \left(\frac{Q}{\pi R}\right) \Delta s = \frac{1}{4\pi\epsilon_0} \frac{Q}{\pi R^2} \sum_i R\Delta\theta = \frac{1}{4\pi\epsilon_0} \frac{Q}{\pi R} \sum_i \Delta\theta.$$

All of the terms come to the front of the summation because these quantities did not change as far as the summation is concerned. The summation does not have to convert to an integral because the sum of all the  $\Delta\theta$  around the semicircle is  $\pi$ . Hence, the potential at the center of a charged semicircle is

$$V_{\text{center}} = \frac{1}{4\pi\epsilon_0} \frac{Q\pi}{\pi R} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$