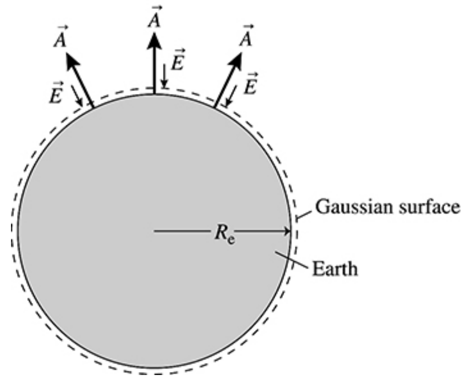
28.41. Model: The charge distribution at the surface of the earth is assumed to be uniform and to have spherical symmetry.

Visualize:

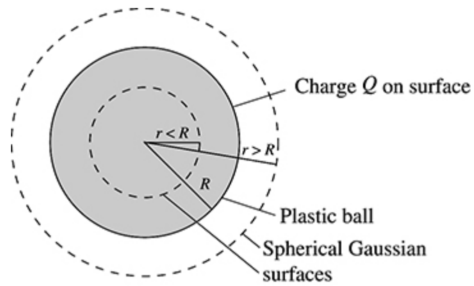


Due to the symmetry of the charge distribution, $\vec{E}$ is perpendicular to the Gaussian surface and the field strength has the same value at all points on the surface.

Solve: Gauss's law is $\Phi_e = \oint \vec{E} \cdot d\vec{A} = Q_{in}/\varepsilon_0$. The electric field points inward (negative flux), hence

$$Q_{in} = -\varepsilon_0 E A_{sphere} = -\left(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2\right)\left(100\ \text{N/C}\right)4\pi\left(6.37\times10^{6}\ \text{m}\right)^2 = -4.51\times10^{5}\ \text{C}$$

28.45. Model: The hollow plastic ball has a charge uniformly distributed on its outer surface. This distribution leads to a spherically symmetric electric field.
Visualize:



The figure shows Gaussian surfaces at $r < R$ and $r > R$.
Solve: **(a)** Gauss's law for the Gaussian surface for $r < R$ where $Q_{in} = 0$ is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = 0 \text{ N m}^2/\text{C} \Rightarrow E = 0 \text{ N/C}$$

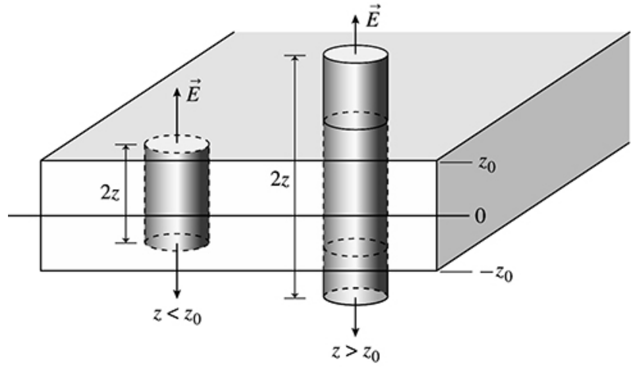
(b) Gauss's law for the Gaussian surface for $r > R$ is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = EA_{sphere} = \frac{Q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \Rightarrow EA_{sphere} = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{\varepsilon_0 A_{sphere}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Assess: A uniform spherical shell of charge has the same electric field at $r > R$ as a point charge placed at the center of the sphere. Additionally, the shell of charge exerts no electric force on a charged particle inside the shell.

28.48. Model: The charge has planar symmetry, so the electric field must point toward or away from the slab. Furthermore, the field strength must be the same at equal distances on either side of the center of the slab.

Visualize:



Choose Gaussian surfaces to be cylinders of length $2z$ centered on the $z = 0$ plane. The ends of the cylinders have area A.

Solve: **(a)** For the Gaussian cylinder inside the slab, with $z < z_0$, Gauss's law is

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} + \int_{\text{sides}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

The field is parallel to the sides, so the third integral is zero. The field emerges from both ends, so the first two integrals are the same. The charge enclosed is the volume of the cylinder multiplied by the charge density, or $Q_{\text{in}} = \rho V = \rho(2zA)$. Thus

$$\frac{Q_{\text{in}}}{\varepsilon_0} = \frac{\rho(2zA)}{\varepsilon_0} = \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} + 0 = 2EA \Rightarrow E = \frac{\rho z}{\varepsilon_0}$$

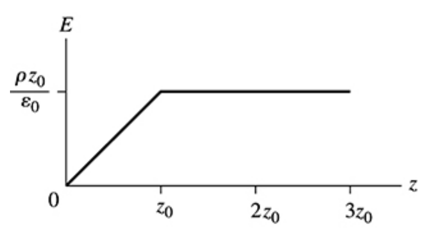
The field increases linearly with distance from the center.

(b) The analysis is the same for the cylinder that extends outside the slab, with $z > z_0$, except that the enclosed charge $Q = \rho(2z_0A)$ is that within a cylinder of length $2z_0$ rather than $2z$. Thus

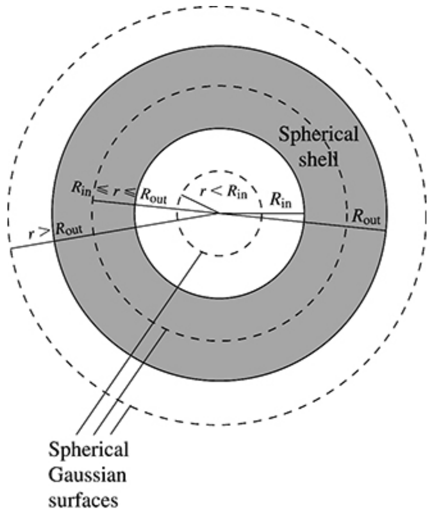
$$\frac{Q_{\text{in}}}{\varepsilon_0} = \frac{\rho(2z_0A)}{\varepsilon_0} = \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} + 0 = 2EA \Rightarrow E = \frac{\rho z_0}{\varepsilon_0}$$

The field strength outside the slab is constant, and it matches the result of part (a) at the boundary.

(c)



28.53. Model: The charge distribution in the shell has spherical symmetry.
Visualize:



The spherical surfaces of radii $r \geq R_{out}$, $r \leq R_{in}$, and $R_{in} \leq r \leq R_{out}$, concentric with the spherical shell, are Gaussian surfaces.

Solve: **(a)** Gauss's law for the Gaussian surface $r \geq R_{out}$ is

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \Rightarrow E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

The vector form comes from the fact that the field is directed radially outward.

(b) For $r \leq R_{in}$, Gauss's law is

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = \frac{0\text{ C}}{\varepsilon_0} \;\Rightarrow\; \vec{E} = \vec{0}$$

(c) For $R_{in} \leq r \leq R_{out}$, Gauss's law is

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \Rightarrow E\left(4\pi r^2\right) = \frac{Q_{in}}{\varepsilon_0}$$

$$Q_{in} = \frac{4\pi}{3}\left(r^3 - R_{in}^3\right)\rho = \frac{4\pi}{3}\frac{\left(r^3 - R_{in}^3\right)Q}{\frac{4\pi}{3}\left(R_{out}^3 - R_{in}^3\right)} = Q\left(\frac{r^3 - R_{in}^3}{R_{out}^3 - R_{in}^3}\right)$$

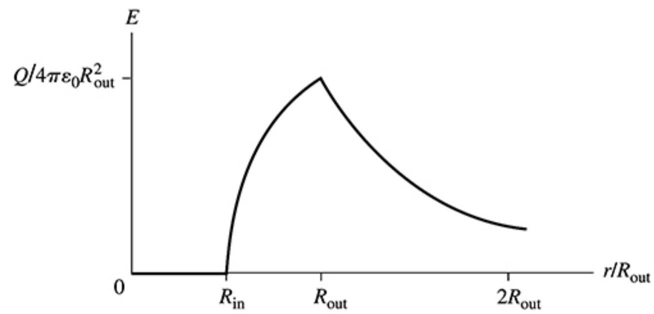
$$\Rightarrow E = \frac{1}{4\pi r^2}\frac{Q}{\varepsilon_0}\frac{r^3 - R_{in}^3}{R_{out}^3 - R_{in}^3} \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(\frac{r^3 - R_{in}^3}{R_{out}^3 - R_{in}^3}\right)\hat{r}$$

(d) The result obtained in part (c) for the electric field simplifies to $\vec{E} = \vec{0}$, when $r = R_{in}$ which is the result obtained in part (b). Furthermore, at $r = R_{out}$, the electric field obtained in part (c) becomes

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_{out}^2}\hat{r}$$

which is the same as the electric field obtained in part (a).

(e)


E
Q/4πε0R2out
0
Rin
Rout
2Rout
r/Rout