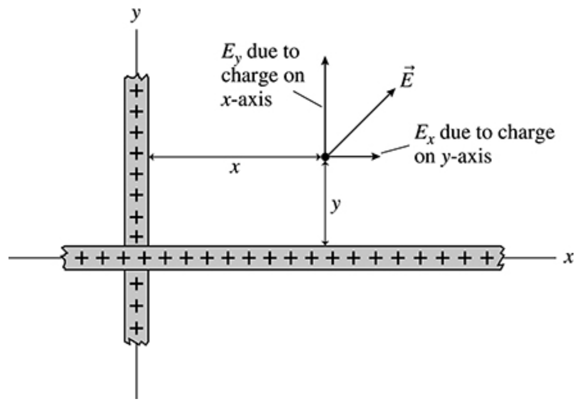
27.38. Model: The fields are those of two infinite lines of charge with linear charge density λ.
Visualize:



Solve: The field at distance r from an infinite line of charge is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}\hat{r}$$

It points straight away from the line. With two perpendicular lines, the field due to the line along the x-axis points in the y-direction and depends inversely on distance y. Similarly, the field due to the line along the y-axis points in the x-direction and depends inversely on distance x. That is

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{x} \qquad E_y = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{y}$$

$$\Rightarrow \vec{E} = E_x\hat{i} + E_y\hat{j} = \frac{2\lambda}{4\pi\varepsilon_0}\left(\frac{1}{x}\hat{i} + \frac{1}{y}\hat{j}\right)$$

The field strength at this point in space is

$$E = \sqrt{E_x^2 + E_y^2} = \frac{2\lambda}{4\pi\varepsilon_0}\sqrt{1/x^2 + 1/y^2}$$

27.45. Model: Assume that the ring of charge is thin and that the charge lies along circle of radius R.
Solve: **(a)** From Example 27.5, the on-axis field of a ring of charge Q and radius R is

$$E_z = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{\left(z^2+R^2\right)^{3/2}}$$

For the field to be maximum at a particular value of z, $dE/dz = 0$. Taking the derivative,

$$\frac{dE}{dz} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\left(z^2+R^2\right)^{3/2}} - \frac{z(3/2)(2z)}{\left(z^2+R^2\right)^{5/2}}\right] = 0 \Rightarrow \frac{1}{\left(z^2+R^2\right)^{3/2}} = \frac{3z^2}{\left(z^2+R^2\right)^{5/2}}$$

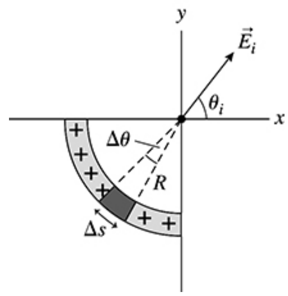
$$\Rightarrow 1 = \frac{3z^2}{z^2+R^2} \Rightarrow z = \frac{R}{\sqrt{2}}$$

(b) The field strength at the point $z = R/\sqrt{2}$ is

$$\left(E_z\right)_{\max} = \frac{Q}{4\pi\varepsilon_0}\frac{\left(R/\sqrt{2}\right)}{\left[\left(R/\sqrt{2}\right)^2 + R^2\right]^{3/2}} = \frac{2}{3\sqrt{3}}\frac{Q}{4\pi\varepsilon_0 R^2}$$

27.47. Model: Assume that the quarter-circle plastic rod is thin and that the charge lies along the quarter-circle of radius R.

Visualize:



The origin of the coordinate system is at the center of the circle. Divide the rod into many small segments of charge Δq and arc length Δs.

Solve: (a) Segment i creates a small electric field $\vec{E}_i$ at the origin with two components:

$$(E_i)_x = E_i\cos\theta_i \qquad (E_i)_y = E_i\sin\theta_i$$

Note that the angle θ_i depends on the location of the segment i. Now all segments are at distance $r_i = R$ from the origin, so

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r_i^2} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}$$

The linear charge density of the rod is $\lambda = Q/L$, where L is the rod's length (L = quarter-circumference = $\pi R/2$). This allows us to relate charge Δq to the arc length Δs through

$$\Delta q = \lambda\Delta s = \left(\frac{Q}{L}\right)\Delta s = \left(\frac{2Q}{\pi R}\right)\Delta s$$

Using $\Delta s = R\Delta\theta$, the components of the electric field at the origin are

$$(E_i)_x = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}\cos\theta_i = \frac{1}{4\pi_0}\frac{1}{R^2}\left(\frac{2Q}{\pi R}\right)R\Delta\theta\cos\theta_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\Delta\theta\cos\theta_i$$

$$(E_i)_y = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}\sin\theta_i = \frac{1}{4\pi\varepsilon_0}\frac{1}{R^2}\left(\frac{2Q}{\pi R}\right)R\Delta\theta\sin\theta_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\Delta\theta\sin\theta_i$$

(b) The x- and y-components of the electric field for the entire rod are the integrals of the expressions in part (a) from

$\theta = 0$ rad to $\theta = \pi/2$. We have

$$E_x = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\int_0^{\pi/2}\cos\theta\, d\theta \qquad E_y = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\int_0^{\pi/2}\sin\theta\, d\theta$$

(c) The integrals are

$$\int_0^{\pi/2}\sin\theta d\theta = [-\cos\theta]_0^{\pi/2} = -\left(\cos\frac{\pi}{2} - \cos 0\right) = +1 \qquad \int_0^{\pi/2}\cos\theta d\theta = [\sin\theta]_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = +1$$

The electric field is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\pi R^2}\left(\hat{i} + \hat{j}\right)$$

27.53. Model: Assume that the electric field inside the capacitor is constant, so constant-acceleration kinematic equations apply.
Visualize: Please refer to Figure P27.53.
Solve: **(a)** The force on the electron inside the capacitor is

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

Because $\vec{E}$ is directed upward (from the positive plate to the negative plate) and $q = -1.60\times10^{-19}$ C , the acceleration of the electron is downward. We can therefore write the above equation as simply $a_y = qE/m$. To determine E, we must first find a_y. From kinematics,

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2 \Rightarrow 0.040 \text{ m} = 0 \text{ m} + v_0 \cos 45°(t_1 - t_0) + 0 \text{ m}$$

$$\Rightarrow (t_1 - t_0) = \frac{(0.040 \text{ m})}{(5.0\times10^6 \text{ m/s})\cos 45°} = 1.1314\times10^{-8} \text{ s}$$

Using the kinematic equations for the motion in the y-direction,

$$v_{1y} = v_{0y} + a_y\left(\frac{t_1 - t_0}{2}\right) \Rightarrow 0 \text{ m/s} = v_0 \sin 45° + \left(\frac{qE}{m}\right)\left(\frac{t_1 - t_0}{2}\right)$$

$$\Rightarrow E = -\frac{2 \text{ m } v_0 \sin 45°}{q(t_1 - t_0)} = -\frac{2(9.1\times10^{-31} \text{ kg})(5.0\times10^6 \text{ m/s})\sin 45°}{(-1.60\times10^{-19} \text{ C})(1.1314\times10^{-8} \text{ s})} = 3550 \text{ N/C} = 3.6\times10^3 \text{ N/C}$$

(b) To determine the separation between the two plates, we note that $y_0 = 0$ m and $v_{0y} = (5.0\times10^6 \text{ m/s})\sin 45°$, but at $y = y_1$, the electron's highest point, $v_{1y} = 0$ m/s. From kinematics,

$$v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0) \Rightarrow 0 \text{ m}^2/\text{s}^2 = v_0^2 \sin^2 45° + 2a_y(y_1 - y_0)$$

$$\Rightarrow (y_1 - y_0) = -\frac{v_0^2 \sin^2 45°}{2a_y} = -\frac{v_0^2}{4a_y}$$

From part (a),

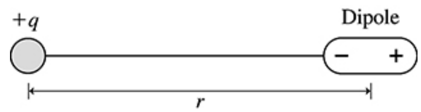
$$a_y = \frac{qE}{m} = \frac{(-1.60\times10^{-19} \text{ C})(3550 \text{ N/C})}{9.1\times10^{-31} \text{ kg}} = -6.242\times10^{14} \text{ m/s}^2$$

$$\Rightarrow y_1 - y_0 = -\frac{(5.0\times10^6 \text{ m/s})^2}{4(-6.242\times10^{14} \text{ m/s}^2)} = 0.010 \text{ m} = 1.0 \text{ cm}$$

This is the height of the electron's trajectory, so the minimum spacing is 1.0 cm.

27.60. Model: The electric field at the dipole's location is that of the ion with charge q.

Visualize:



Solve: **(a)** We have $p = \alpha E$. The units of α are the units of p/E and are

$$\frac{\text{C m}}{\text{N/C}} = \frac{\text{C}^2\ \text{m}}{\text{N}} = \frac{\text{C}^2\text{s}^2}{\text{kg}}$$

(b) The electric field due to the ion at the location of the dipole is

$$E_{\text{at dipole}} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2},\ \text{away from } q\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{i}$$

Because $\vec{p} = \alpha\vec{E}$, the induced dipole moment is

$$\vec{p} = \alpha\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\hat{i}$$

From Equation 27.11, the electric field produced by the dipole at the location of the ion is

$$\vec{E}_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2}{r^3}\right)\alpha\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\hat{i} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{2q\alpha}{r^5}\right)\hat{i}$$

The force the dipole exerts on the ion is

$$\vec{F}_{\text{dipole on ion}} = q\vec{E}_{\text{dipole}} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{2q^2\alpha}{r^5}\right)\hat{i}$$

According to Newton's third law, $\vec{F}_{\text{dipole on ion}} = -\vec{F}_{\text{ion on dipole}}$. Therefore,

$$\vec{F}_{\text{ion on dipole}} = \left(\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{2q^2\alpha}{r^5},\ \text{toward ion}\right)$$