1	2	3	4	Total

Phy212 Electromagnetism Spring 2009

Mid-term examination II

Please print your name and circle your workshop section number

Name________________________________

Workshop Sec. #

M002	M003	M004	M005	M007	M009
Bunk	Hubisz	Degaga	Degaga	Degaga	Bunk

Please read the following sentence and sign your name below

By submitting this exam, I pledge that I will not give or receive any unauthorized assistance.

Signature________________________________ Date__________

Problem 1 (Conceptual questions, 40 points total):

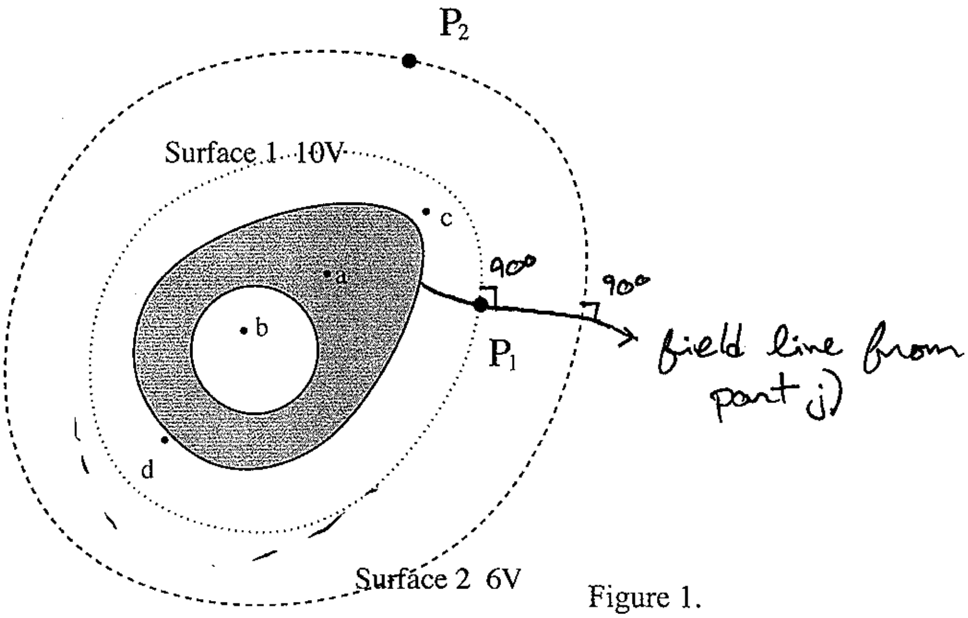


Figure 1.

A hollow conductor has a cavity inside (white region) and carries a total charge of $Q > 0$, as shown in the figure. The conductor is insulated from the rest of the world. Answer the following questions. (No explanation is required)

a) (3pts) What is the total charge on the inner surface of the cavity?

$\boxed{Q_{in}=0}$:  gaussian surface; $E=0$ inside conductor

$\Phi = \int \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = 0$

b) (2pts) What is the electric field at point a?

$\vec{E}_a = 0$ (otherwise current induced $\Rightarrow$ not electrostatic equilibrium!)

c) (2pts) What is the electric field at point b?

$\vec{E}_b = 0$ again using Gauss' law

d) (2pts) Which one is stronger, the electric field at point c or the electric field at point d?

$|\vec{E}_c| > |\vec{E}_d|$ Charge clusters more at sharper end, also equipotential lines are more dense

e) (3pts) Let Va, Vb, Vc be the electric potential at points a, b, c respectively. List the relative order between them: (i.e. which one is larger than which one, etc.)

$V_a = V_b > V_c$

f) (3pts) Now consider the equi-potential surface 1 (dotted) shown in the figure. What is the integral of the electric field over the surface 1?

$$\oint_{\text{surface 1}} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

g) (3pts) Suppose the potential is 10V on the surface 1, and 6V on the surface 2, what is the work done by the electric field if a charge q moves from point P_1 to point P_2?

$$W = -q\Delta V = -q(6V - 10V) = (4V)q$$

h) (3pts) What is the potential at a point that is distance R away from the conductor? Suppose R is much larger than the conductor size and there is no other charge in the world.

Far away, looks like point charge!

$$V = \frac{Q}{4\pi \varepsilon_0 R}$$

i) (3pts) If we release a particle with charge $q > 0$ at rest at point P_1, the charge will accelerate and move away from the conductor. What is its velocity when the charge is infinitely far away?

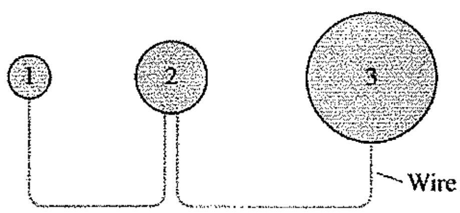
at $R \to \infty$ $V = 0$ | $-(\Delta V)q = \frac{1}{2}mv^2$ and $-\Delta V = 10V$

$$v = \left[\frac{2(10V)q}{m}\right]^{1/2}$$

j) (3pts) Draw a field line passing through point P_1, indicate the angle between the field line and the equi-potential surface.

see figure

Three conductor spheres of different sizes are connected to each other by conducting wires. Let V_1, V_2, V_3 be the potentials on the three spheres, while E_1, E_2, E_3 be the electric fields right outside three spheres.

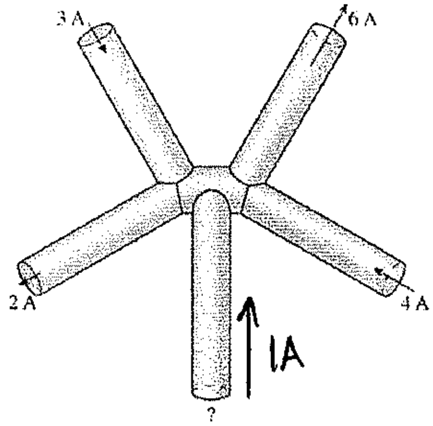


k) (2pts) List the relative order between V_1, V_2, V_3

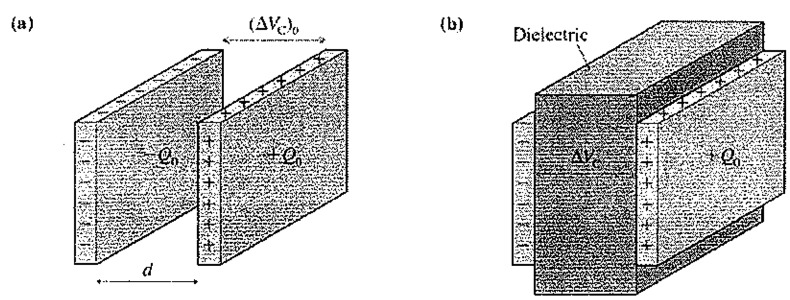
$$V_1 = V_2 = V_3$$

l) (2pts) List the relative order between E_1, E_2, E_3

$$E_1 > E_2 > E_3$$

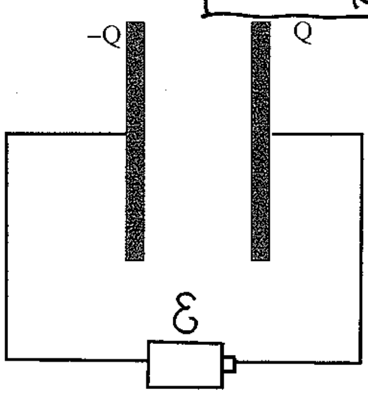


m) (3pts) A junction with five branches in certain circuit is shown above. Show in the figure the direction and magnitude of the current on the center branch with question mark.



n) (3pts) See the figure above. A parallel plate capacitor carries charge Q_0 in the left. Now consider inserting a chunk of dielectric materials with dielectric constant $\kappa > 1$, how does the voltage between two plates change?

$$V_f = \frac{V_i}{\kappa}$$

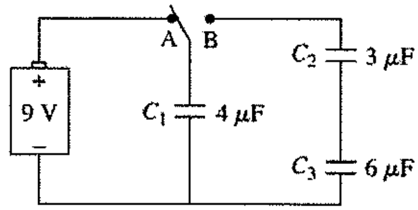


o) (3pts) Now consider a parallel plate capacitor that is connected to a battery with EMF shown in the figure. As we insert a chunk of dielectrics with dielectric constant $\kappa > 1$ between two plates, how does the charge on the capacitor change?

$$Q_f = \kappa Q_i$$

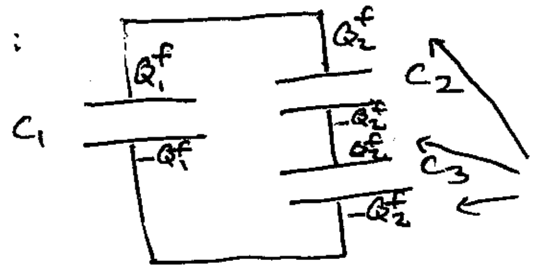
Problem 2 (20 points)

Initially the switch is in position A and capacitors C_2 and C_3 are uncharged. After the switch is flipped to position B, what are the charge on and potential difference across each capacitor?



First find Q_1^i: $Q_1^i = C_1 V = (4\times10^{-6}F)(9V) = 36\times10^{-6}C$

Final circuit:



$Q_1^f + Q_2^f = 36\times10^{-6}C$ and all are Q_2^f by conservation of charge $(Q_2^f = Q_3^f)$

now there is no EMF, so

$$V_1 + V_2 + V_3 = \frac{Q_1^f}{C_1} - \frac{Q_2^f}{C_2} - \frac{Q_2^f}{C_3} = 0$$

and $Q_1^f = 36\times10^{-6}C - Q_2^f$

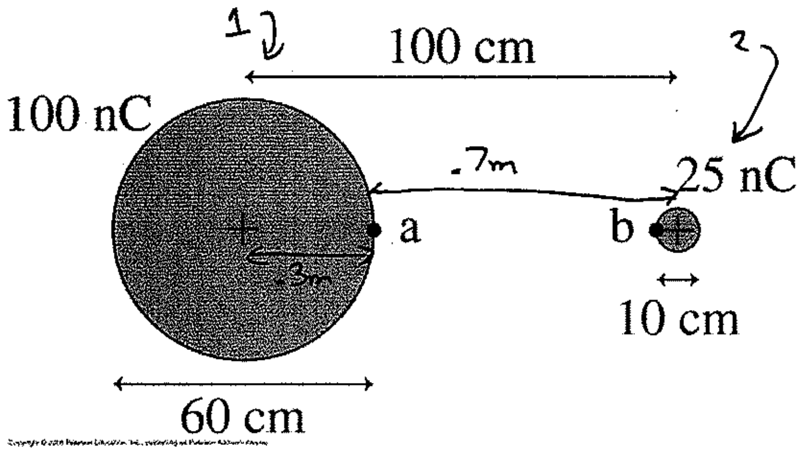
$$\Rightarrow \frac{36\times10^{-6}C}{4\mu F} - Q_2^f\left(\frac{1}{4\mu F} + \frac{1}{3\mu F} + \frac{1}{6\mu F}\right) = 0$$

$$\Rightarrow \boxed{Q_2^f = 12\times10^{-6}C}$$

$$\boxed{Q_1^f = 24\times10^{-6}C}$$

$$\boxed{V_1 = \frac{Q_1^f}{C_1} = 6V} \quad \boxed{V_2 = \frac{Q_2^f}{C_2} = 4V} \quad \boxed{V_3 = \frac{Q_3^f}{C_3} = 2V}$$

Problem 3 (20 points)



The figure above shows two uniformly charged spheres. The distance between the centers of two spheres is 100cm, s shown in the figure.

a) (10pts) What is the potential at point a? (Hint: The potential at any point is the superposition of the potentials due to all charges.)

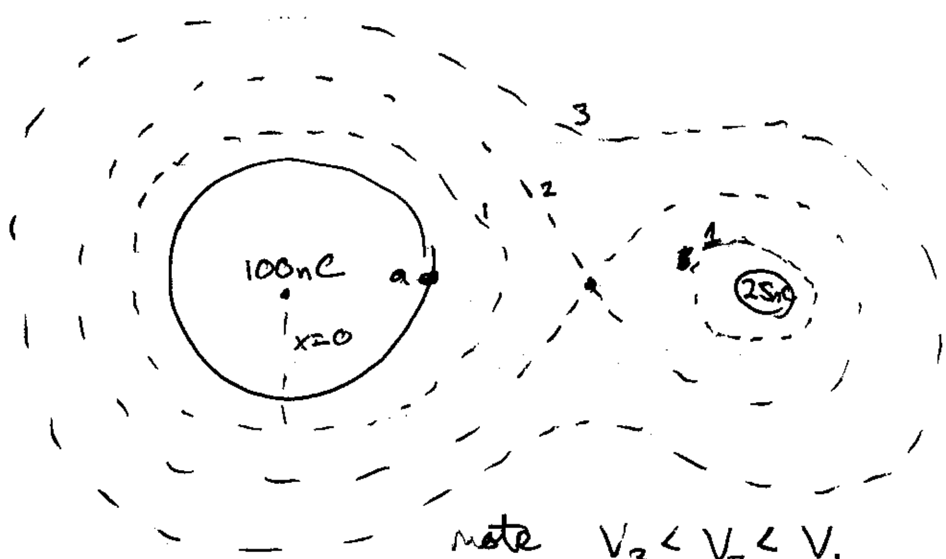
$$V_a = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{100\times10^{-9}C}{.30m} + \frac{25\times10^{-9}C}{.70m}\right]$$

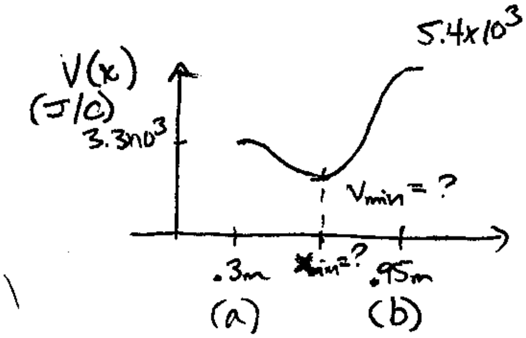
$$\boxed{V_a = 3.3\times10^{3}\ V}$$

b) (10pts) With two spheres fixed in space, suppose we release a mass point with negative charge -10 nC and mass m at point a. What is the minimum initial velocity of the mass point so that it can reach point b?

Need to find U as a function of position

Equipotential lines:





note $V_3 < V_2 < V_1$

2pt conservation

and $U = (-10nC)V$, so $U_3 > U_2 > U_1$

→ best way to go is straight across

at minimum of potential between spheres, $\frac{\partial V}{\partial x} = 0$ (or $\vec{E} = 0$)

$$V_{tot}(x) = \frac{1}{4\pi\varepsilon_0} 25\times10^{-9}C \left[\frac{4}{x} + \frac{1}{(1m - x)} \right]$$

$$\frac{\partial V}{\partial x} \propto \frac{4}{x^2} - \frac{1}{(1m - x)^2} = 0 \Rightarrow$$ x_{max} = [redacted] .67m

$$U(x_{max}) = + \frac{(-10nC)(25\times10^{-9}C)}{4\pi\varepsilon_0}\left[\frac{4}{.67m} + \frac{1}{.33m}\right] = -2\times10^{-5}\,J$$

$$U_a = +qV_a = -(10\times10^{-9}C)(3.3\times10^{3}V) = -3.3\times10^{-5}\,J$$

$$\Delta U = 1.3\times10^{-5}\,J = \frac{1}{2}mv^2 \Rightarrow \boxed{v = \frac{5.1\times10^{-3}(m/s)\cdot kg^{1/2}}{\sqrt{m}}}$$

Problem 4 (20points)

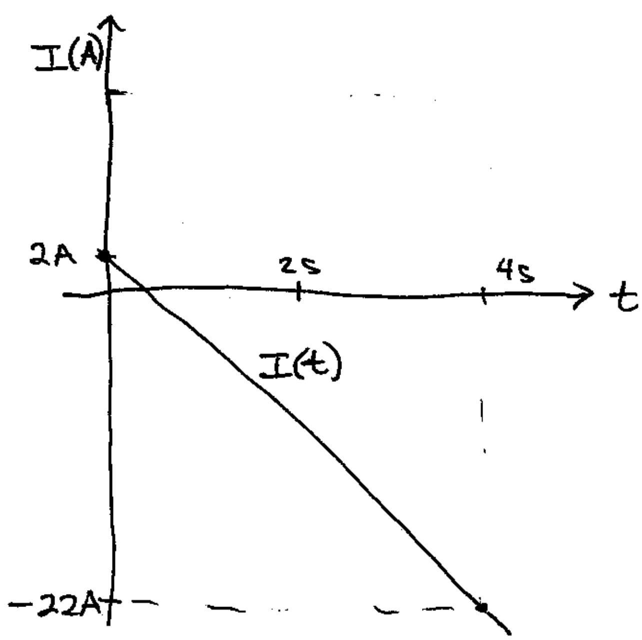
Part one:

The total amount of charge in coulombs that has entered a wire at time t is given by the expression $Q = 2\,t - 3\,t^2$, where t is in seconds.

a) (5pts) Find an expression for the current in the wire at time t

$$I = \frac{dQ}{dt} = 2 - 6t$$

b) (5pts) Graph I versus t for the interval $0<t<4$ s.



Part two

A 30 cm long copper wire with diameter 2mm is connected to a battery with EMF 3.0V.

a) (5pts) What is the electric field inside the wire?

$$\vec{E} = -\frac{\partial V}{\partial x} = -\frac{\Delta V}{\Delta x} \Rightarrow \boxed{|\vec{E}| = \frac{3.0V}{.3m} = 10\ V/m}$$

b) (5pts) What is the current density inside the wire?

$$V = IR = I\frac{\rho l}{A} = \frac{I}{A}\rho l \qquad \text{and} \quad J = \frac{I}{A}$$

(l ← length of wire; ρ ← resistivity; A ← x-sec. area of wire)

$$V = J\rho l\ ; \quad J = \frac{V}{\rho l} = \frac{3.0V}{\rho(.3m)} = \frac{10\ V/m}{\rho}$$

$$\left(\text{or } J = \frac{|\vec{E}|}{\rho}\right)$$

Note: for copper, $\rho = 1.7\times10^{-8}\ \Omega\text{-m}$

$$\Rightarrow \boxed{J = 5.9\times10^{8}\ A/m^2}$$