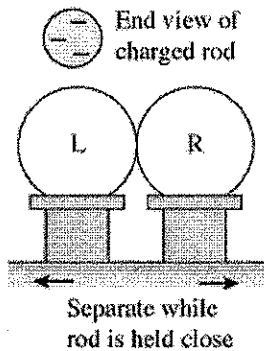


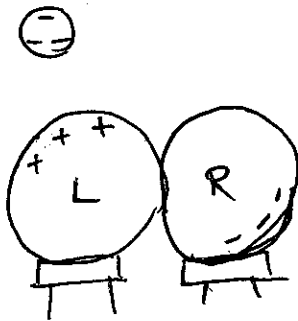
Problem 1 (Conceptual questions, 30 points total):

- a) (5 points) Two neutral metal spheres on insulating stands are placed in contact. A negatively charged rod is then brought directly over the top of the left sphere, as shown, but does not touch either sphere. While the rod is held near, the spheres are separated slightly so that they no longer touch. Then the charged rod is withdrawn. After the rod is withdrawn, what is the charge state (positive, negative, or neutral) of each sphere? Use both words *and* charge diagrams to explain your answer.



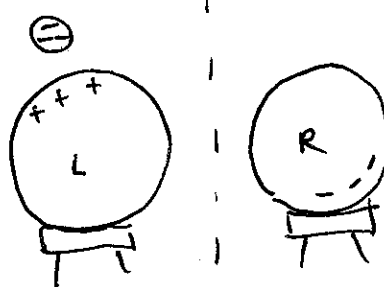
Charge diagrams

I.



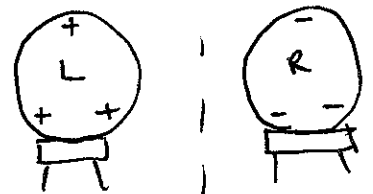
The charged rod polarizes the conductors: + on L - on R. (- rod repels electrons in conductor)

II



Spheres are separate but charges remain where they are

far apart

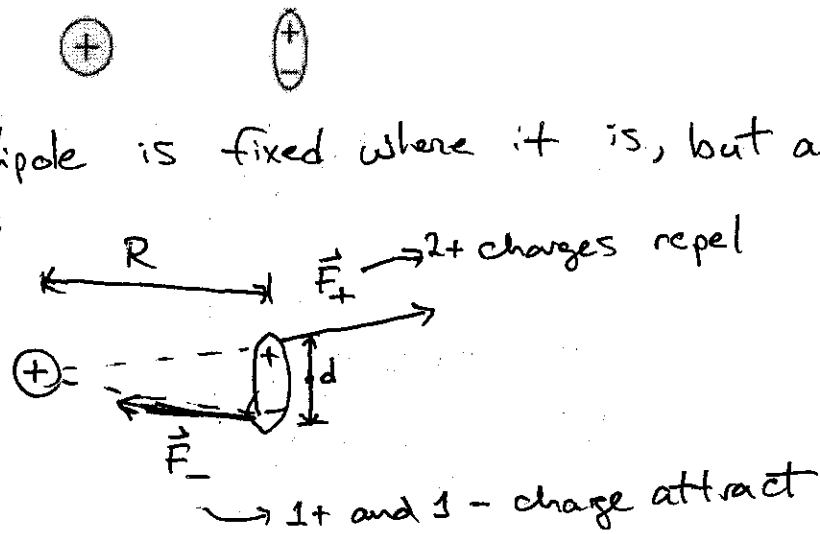


Rod removed and spheres moved far apart. Excess charge distributed uniformly on surfaces of each.

Ans: L is + charged, R is - charged

b) (5 points) A positive point charge is brought near a permanent electric dipole that is oriented as shown. Describe and explain how the dipole responds.

Assuming the dipole is fixed where it is, but allowed to rotate:

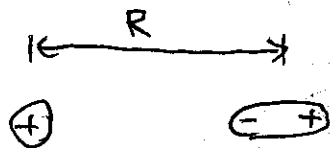


for large R \vec{F}_+ creates torque $|\vec{\tau}_+| = |\vec{F}_+| \frac{d}{2}$ (and $|\vec{F}_+| = |\vec{F}_-|$)
 \vec{F}_- " " $|\vec{\tau}_-| = |\vec{F}_-| \frac{d}{2}$

torque vectors add constructively (don't cancel)

so there is net torque $|\vec{\tau}_{net}| = |\vec{F}_\pm| d$

so dipole rotates clockwise with equilibrium position!



c) (5 points) A positively charged glass rod is held over a thin, uncharged piece of metal foil. How does the foil respond? Give a step-by-step explanation, using both words *and* charge diagrams.

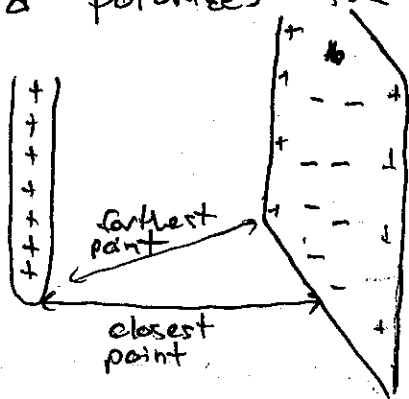


Charged rod

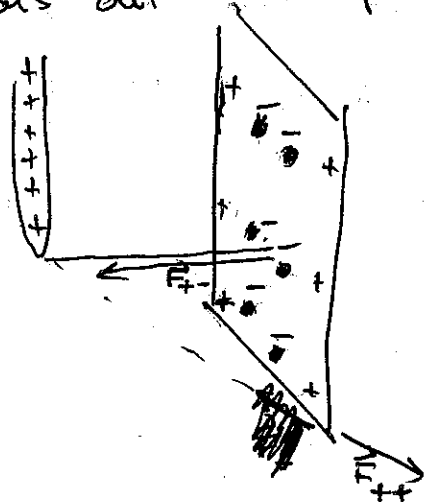


Thin foil

I. The charged polarizes the thin foil: ← a conductor!

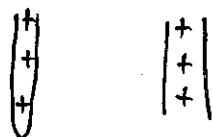


II + attracts - and + repels +, but + charges are farther away from rod. Thus the attractive force beats out the repelling force.



So the foil is attracted to the rod

III After they touch: foil gains + charge and they repel each other!



d) (10 points)

i) (3pts) How is a conductor different from an insulator?

In a conductor, there is a valance band of electrons that are free to move among the atoms of ~~the~~ it. Insulators' electrons are all tightly bound, and fixed near their parent nuclei.

ii) (2pts) Will a charged plastic rod attracts small pieces of paper (insulator)?

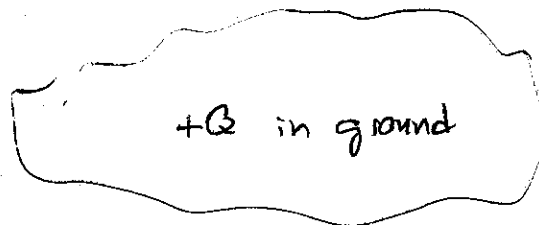
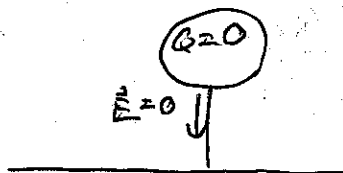
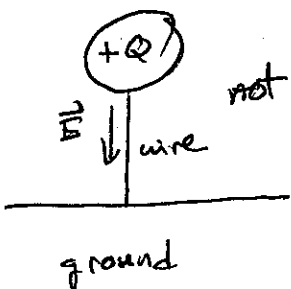
Yes, due to the fact that the constituent atoms of the insulator are polarized. Charged objects attract polarized ones.

iii) (2pts) Will the rod attract small pieces of aluminum foil (conductor)?

Yes, see solution to part c).

iv) (3pts) What happens when you ground a charged metal plate?

The "ground" is an infinite size conductor. To ground an object is to connect it to the ground via a wire (or other conductor). To eliminate \vec{E} in the wire, charge leaves from the charged metal to ground. Since ground is infinite (for all practical purposes) residual charge on object is zero.



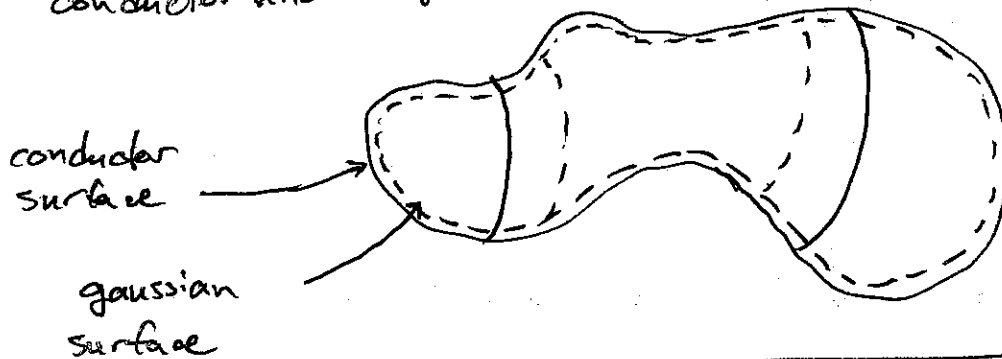
e) (5 points) Explain in words

i) Why is it that the electric field inside any conductor must vanish?

If $\vec{E}_{\text{conductor}} \neq 0$, then electrons will move in a direction: $-\frac{\vec{E}_{\text{conductor}}}{|\vec{E}_{\text{conductor}}|}$ thus $\vec{E} \neq 0$ is a non-equilibrium configuration.

ii) Why is that there can be no net charge inside any conductor?

From part i) we know $\vec{E}_{\text{cond}} = 0$. Look at an arbitrary conductor and imagine gaussian surface arbitrarily close to surface of conductor



Gauss' Law: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$

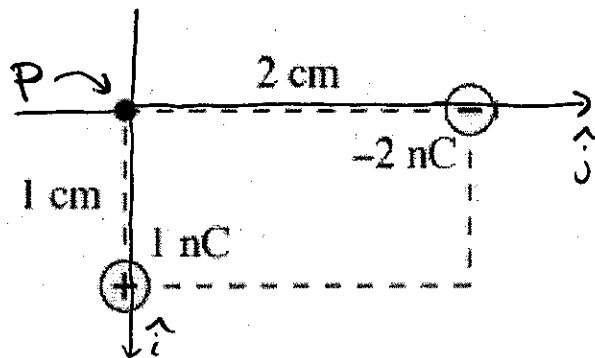
↑
int. over
gaussian
surface

since $\vec{E} = 0$ inside,

$$\frac{Q_{\text{enc}}}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A} = \oint \overset{\text{zero!}}{\vec{0}} \cdot d\vec{A} = 0$$

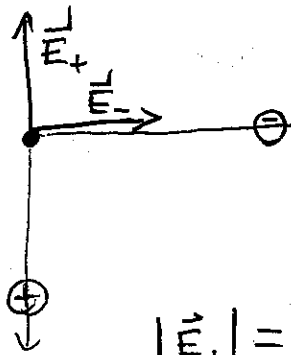
Problem 2 (25 points)

Consider two charges shown in the figure below and the point P.



- a) (13 pts) What is the electric field at the point indicated as a dot? Express your answer in component form.

at P:



\vec{E}_+ is in $-\hat{i}$ direction
 \vec{E}_- " " $+\hat{j}$ direction

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$|\vec{E}_+| = \frac{1}{4\pi\epsilon_0} \frac{(1 \times 10^{-9} \text{ C})}{(0.01 \text{ m})^2} = 9 \times 10^4 \text{ N/C}$$

$$|\vec{E}_-| = \frac{1}{4\pi\epsilon_0} \frac{(2 \times 10^{-9} \text{ C})}{(0.02 \text{ m})^2} = 4.5 \times 10^4 \text{ N/C}$$

$$\vec{E}_{\text{tot}} = \vec{E}_+ + \vec{E}_- = 9 \times 10^4 \text{ N/C } (-\hat{i}) + 4.5 \times 10^4 \text{ N/C } (+\hat{j})$$

or $\vec{E}_{\text{tot}} = (-9 \times 10^4 \text{ N/C}, 4.5 \times 10^4 \text{ N/C})$

b) (12pts) What is the acceleration of an electron at that point? Give your answer as a magnitude and a direction.

force on charge is $\vec{E} q_{e^-} = \vec{F} = m_{e^-} \vec{a}$

↑ charge of electron

↑ mass of electron

$$q_{e^-} = -1.6 \times 10^{-19} \text{ C}$$

$$m_{e^-} = 9.1 \times 10^{-31} \text{ kg}$$

$$\vec{a} = \frac{\vec{E} q_{e^-}}{m_{e^-}} = \frac{(-9 \times 10^4 \text{ N/C}, 4.5 \times 10^4 \text{ N/C})(-1.6 \times 10^{-19} \text{ C})}{(9.1 \times 10^{-31} \text{ kg})}$$

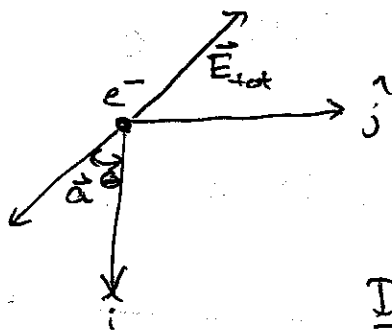
$$= (+1.6 \times 10^{16} \text{ m/s}^2, -7.9 \times 10^{15} \text{ m/s}^2)$$

↓ a_x a_y

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = [(1.6 \times 10^{16} \text{ m/s}^2)^2 + (7.9 \times 10^{15} \text{ m/s}^2)^2]^{1/2}$$

Magnitude:

$$|\vec{a}| = 1.8 \times 10^{16} \text{ m/s}^2$$



$$\tan \theta = \left| \frac{a_y}{a_x} \right| \Rightarrow \theta = \tan^{-1} \left(\frac{.79}{1.6} \right) = 26.6^\circ$$

Direction:

26.6° clockwise from $+\hat{i}$ unit vector

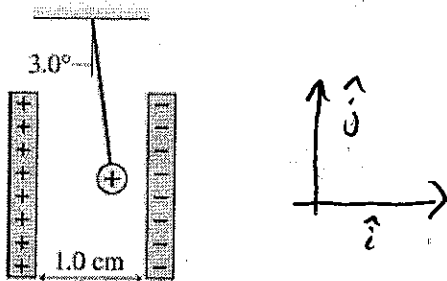
or

63.4° counter-clockwise from $-\hat{j}$ unit vector

etc...

Problem 3 (25 points)

A 100mg pith ball hangs by a thread in the center of a parallel-plate capacitor. The capacitor plates are 5.0cm X 5.0cm squares and are spaced 1.0cm apart. The capacitor plates are charged to +Q and -Q respectively.



- a) (5ps) What is the surface charge density on two plates? Express your answer in terms of numbers and the unknown variable Q.

$$\eta = \frac{Q}{\text{Area}} = \frac{Q}{L^2} = \frac{Q}{(.05\text{m})^2} = (400\text{m}^{-2})Q$$

- b) (5pts) What is the electric field between two plates? Again express your answer in terms of numbers and the unknown variable Q.

for parallel plates

$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, + \text{to } - \right)$$

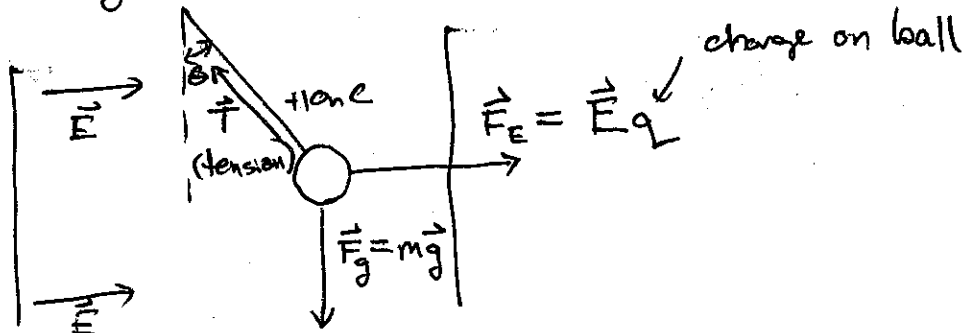
\hat{i} in picture above

$$\vec{E} = \frac{1}{\epsilon_0} \frac{Q}{(.05\text{m})^2} \hat{i} = \left(4.5 \times 10^{13} \frac{\text{N}}{\text{C}^2} \right) Q \hat{i}$$

- c) (15pts) Now consider the ball with charge $+10\text{nC}$ hanging between two plates at a 3.0° angle, as shown in the figure. Find the charge Q .

mass: $m = 100\text{mg} = .1\text{g} = .0001\text{kg} = 10^{-4}\text{kg}$

Bring balance to the Force!



$$|T| \sin \theta = |E| q \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{divide 2 equations}$$

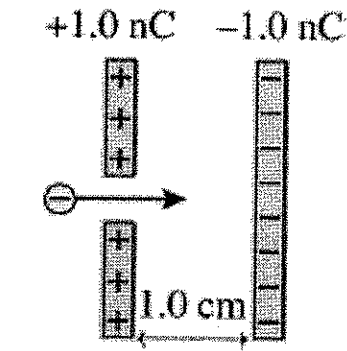
$$|T| \cos \theta = m |g| \quad \left. \begin{array}{l} \\ \end{array} \right\} \frac{|T| \sin \theta}{|T| \cos \theta} = \tan \theta = \frac{|E| q}{m |g|} = \frac{1}{\epsilon_0} \frac{Q}{L^2} \frac{q}{m g}$$

$$Q = \frac{\epsilon_0 \tan \theta L^2 m g}{q} = \frac{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}) \tan(3^\circ) (.05\text{m})^2 (1 \times 10^{-4}\text{kg}) (9.8 \frac{\text{m}}{\text{s}^2})}{(10 \times 10^{-9}\text{C})}$$

$$Q = 1.1 \times 10^{-10} \text{C} = .11 \text{nC}$$

Problem 4 (20 points)

A parallel-plate capacitor is formed of two $10 \text{ cm} \times 10 \text{ cm}$ plates spaced 1.0 cm apart. The plates are charged to $\pm 1.0 \text{ nC}$. An electron is shot through a very small hole in the positive plate. What is the slowest speed the electron can have if it is to reach the negative plate?



$$|\vec{E}| = \frac{\eta}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{Q}{\epsilon_0 L^2}$$

$$|\vec{F}_E| = |\vec{E}| |q| \quad q_{\text{electron}} = -1.6 \times 10^{-19} \text{ C}$$

Work done by force is $|\vec{F}| \Delta x = W$

kinetic energy

$$KE = \frac{1}{2} m v_0^2$$

mass of electron $9.11 \times 10^{-31} \text{ kg}$

energy is conserved:

$$KE = W$$

initial kinetic energy

work done

$$\frac{1}{2} m v_0^2 = |\vec{F}| \Delta x = |\vec{E}| |q| \Delta x$$

$$= \frac{Q}{\epsilon_0 L^2} |q| \Delta x$$

$$v_0 = \left[\frac{2Q|q|\Delta x}{\epsilon_0 L^2 m} \right]^{1/2}$$

$$= \left[\frac{2(1 \times 10^{-9} \text{ C})(1.6 \times 10^{-19} \text{ C})(.01 \text{ m})}{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2})(.1 \text{ m})^2 (9.11 \times 10^{-31} \text{ kg})} \right]^{1/2}$$

$$= 6.3 \times 10^6 \text{ m/s}$$