

# Impossible Measurements on Quantum Fields\*

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## *Abstract*

It is shown that the attempt to extend the notion of ideal measurement to quantum field theory leads to a conflict with locality, because (for most observables) the state vector reduction associated with an ideal measurement acts to transmit information faster than light. Two examples of such information-transfer are given, first in the quantum mechanics of a pair of coupled subsystems, and then for the free scalar field in flat spacetime. It is argued that this problem leaves the Hilbert space formulation of quantum field theory with no definite measurement theory, removing whatever advantages it may have seemed to possess vis a vis the sum-over-histories approach, and reinforcing the view that a sum-over-histories framework is the most promising one for quantum gravity.

## **1. INTRODUCTION: IDEAL MEASUREMENTS AND QUANTUM FIELD THEORY**

Whatever may be its philosophical limitations, the textbook interpretation of non-relativistic quantum mechanics is probably adequate to provide the quantum formalism with all the predictive power required for laboratory applications. It is also self-consistent in the sense that there exist idealized models of measurements which allow the system-observer boundary to be displaced arbitrarily far in the direction of the observer. And the associated “transformation theory” possesses a certain formal beauty, seemingly realizing the “complementarity principle” in terms of the unitary equivalence of all orthonormal bases. It is therefore natural to try to generalize this semantic framework to relativistic quantum field theory in the hope of learning something new, either from the success or failure of the attempt.

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In fact we will see that the attempt fails in a certain sense; and the way it fails suggests that the familiar apparatus of states and observables must give way to a more spacetime-oriented framework, in which the new physical symmetry implied by the “transformation theory” is lost, and the role of measurement as a fundamental concept is transformed or eliminated. Although such a renunciation of the usual measurement formalism might seem a step backwards for quantum field theory, it can also be viewed as a promising development for quantum gravity. It means that some of the conceptual problems which are normally thought of as peculiar to quantum gravity or quantum cosmology are already present in flat space, where their analysis and resolution may be easier.

The framework we will attempt to generalize is based on the notion of an *ideal measurement*, by which I mean one for which (i) the possible outcomes are the eigenvalues of the corresponding operator, realized with probabilities given by the usual trace-rule, and (ii) the standard *projection postulate* correctly describes the effect of the measurement on the subsequent quantum state. Such a “minimally disturbing measurement” is not only a “detection” but is simultaneously a “preparation” as well and, if one follows the textbook interpretation, it is precisely from this dual character of measurement that the predictive power of the quantum mechanical formalism derives. It is also from this dual aspect that the difficulty in relativistic generalization will arise—a difficulty with so-called superluminal signaling.

Now in non-relativistic quantum mechanics, measurements are idealized as occurring at a single moment of time. Correspondingly the interpretive rules for quantum field theory are often stated in terms of ideal measurements which take place on Cauchy hypersurfaces. However, in the interests of dealing with well-defined operators, one usually thickens the hypersurface, and in fact the most general formulations of quantum field theory assume that there corresponds to any open region of spacetime an algebra of observables which—presumably—can be measured by procedures occurring entirely within that region. (Unlike for the non-relativistic case, however, no fully quantum models of such field measurements have been given, as far as I am aware.)

The statement that “one can measure” a single observable  $A$  associated to a spacetime region  $O$  is fine as far as it goes, and an obvious generalization of the projection postulate can be adopted as part of the definition of such a measurement. But a potential confusion arises as soon as we think of two or more separate measurements being made. In the non-relativistic theory, measurements carry a definite temporal order from which the logical sequence of the associated state-vector reductions is derived; but in Minkowski space, the temporal relationships among regions can be

more complicated, and the rules for “collapsing” the state are not necessarily evident. Nonetheless, I claim that a natural set of rules exists, which directly generalizes the prescription of the non-relativistic theory.

## 2. A RELATIVISTIC PROJECTION POSTULATE

The problem is that these “obvious” rules fail to be consistent with established ideas of causality/locality. Hence, the kind of measurements they envisage can presumably not be accomplished. It would of course be very interesting to try to construct models within quantum field theory, to see what goes wrong, but I will not attempt such a von-Neumann-like analysis here. Also, I will restrict myself to flat-space for definiteness, although nothing would be changed by going over to an arbitrary globally hyperbolic spacetime. Finally it seems most convenient to work in the Heisenberg picture, since the association of field operators to regions of spacetime is most direct in that picture.

With these choices made, let us envisage a (finite) collection of ideal measurements to be performed on some quantum field  $\Phi$ . We are then faced with a collection of regions,  $O_k$  in Minkowski space, and corresponding to each region, we are given an observable  $A_k$ , formed from the restriction of  $\Phi$  to  $O_k$ . Given all this and an initial state  $\rho_0$  specified to the past of all of the  $O_k$ , we may ask for the probability of obtaining any specified set of eigenvalues  $\alpha_k$  of the  $A_k$  as measurement outcomes.

Non-relativistically, we would determine these probabilities by ordering the  $A_k$  in time (say with  $A_1$  preceding  $A_2$  preceding  $A_3 \dots$ ), then using  $\rho_0$  to compute probabilities for the earliest observable  $A_1$ , then “reducing”  $\rho$  conditioned on the eigenvalue  $\alpha_1$ , then using this reduced state to compute probabilities for  $A_2$ , etc. In the special case where each of the  $A_k$  is a projection or “question”  $E_k$ , this procedure results, as is well known, in the remarkably simple expression

$$\langle E_1 E_2 \dots E_{n-1} E_n E_{n-1} \dots E_2 E_1 \rangle \tag{1}$$

for the probability of ‘yes’ answers to all the questions, where expectation in the initial state  $\rho_0$  has been denoted simply by angle brackets, a practice I will adhere to henceforth. (That is,  $\langle A \rangle \equiv \text{trace}(\rho_0 A)$ .)

Now let us return to the relativistic situation. In the special case where the regions  $O_k$  are non-intersecting Cauchy surfaces (or slight thickenings thereof), their time-ordering permits a unique labeling, and the generalization of the above non-relativistic procedure is immediate. For a more general set of regions we may try to foliate the spacetime in such a way that the  $O_k$  *acquire* a well-defined temporal ordering (each  $O_k$  being separated from its predecessor by one of the leaves of the foliation). Obviously

not every labeling of the regions can arise from a foliation by Cauchy surfaces, since no region which comes later with respect to such a foliation can intersect the causal past of one which comes earlier. Indeed this restriction makes sense independent of any choice of foliation, and merely says that if a measurement made in one region can possibly influence the outcome of a measurement made in a second region, then the second measurement should be regarded as taking place “later” than the first.

The labelings of the regions which respect this causal restriction can be described systematically in terms of an order relation “ $\prec$ ” reflecting the possibilities of causal influence among the regions. To define  $\prec$  we merely specify that  $O_j \prec O_k$  iff some point of  $O_j$  causally precedes some point of  $O_k$ . A labeling — or equivalently a linear ordering — of the regions is then *compatible with*  $\prec$  iff  $O_j \prec O_k \Rightarrow j \leq k$ . Of course, it can happen that no such labeling exists, in which case the rules described above admit of no natural generalization. To express the exclusion of such cases in a systematic manner, we may take the *transitive closure* of  $\prec$ , obtaining thereby an extended relation, for which I will use the same symbol  $\prec$ . The condition that compatible labelings exist is then that this extended  $\prec$  be what is called a partial order, that is that it never happen that both  $O_j \prec O_k$  and  $O_k \prec O_j$  for some  $j \neq k$ . When, on the contrary, this does happen, we have the analog of two non-relativistic measurements being simultaneous, which, even non-relativistically, leads to no well-defined probabilities unless the corresponding operators happen to commute. We may exclude such cases, by simply requiring that the regions  $O_k$  be disposed so that no such circularity occurs. In particular overlapping regions are thereby excluded by fiat.

Given the partial order  $\prec$ , it is easy to state the natural generalization of the non-relativistic rules for forming probabilities: We simply extend  $\prec$  to a linear order (as can always be done) and use the rules precisely as they were stated earlier. Unless  $\prec$  happens to be already a linear order, the particular choice of linear extension (or equivalently labeling) is not unique, but this ambiguity is harmless *as long as* the field  $\Phi$  satisfies “local commutativity”, i.e. as long as field observables belonging to spacelike separated regions commute. In particular, when the  $A_k$  are all projection-operators, the formula (1) holds exactly as given above, for any labeling of the regions which is compatible with  $\prec$ .

(It is often objected that the idea of state-vector reduction cannot be Lorentz-invariant, since “collapse” will occur along different hypersurfaces in different rest-frames. However we have just seen that well-defined probability rules can be given without associating the successive collapses to any particular hypersurface. Thus the objection is unfounded to the extent that one regards the projection postulate as nothing more

than a rule for computing probabilities. Of course if one takes the state-vector (or density operator?) itself to be physically real, then the puzzle about “where” it collapses might remain.)

Abstractly considered, the scheme we have just taken the trouble to construct seems impeccable, but in fact it has a problem foreshadowed by our need to take a transitive closure in defining  $\prec$ . The problem is that the state-vector reduction implied by an ideal measurement is non-local in such a way as to transmit observable effects faster than light, something like an EPR experiment gone haywire. If we want to reject such superluminal effects, then we will be forced to exclude the possibility of ideal measurements of most of the spacetime observables we have been contemplating.

### 3. THE CONTRADICTION WITH LOCALITY

In speaking of superluminal effects, the situation I have in mind concerns three regions  $O_1$ ,  $O_2$  and  $O_3$ , situated so that some points of the first precede points of the second, and some points of the second precede points of the third, but all points of the first and third are spacelike separated. In fact, let us specialize  $O_2$  to be a thickened spacelike hyperplane, with  $O_1$  and  $O_3$  being bounded regions to its past and future respectively. The corresponding observables  $A_1$ ,  $A_2$ ,  $A_3$ , I will call ‘ $A$ ’, ‘ $B$ ’ and ‘ $C$ ’ in order to save writing of subscripts. The non-locality (or “acausality”) in question then shows up in the fact that, for generic choices of the observables  $A$ ,  $B$ ,  $C$  and of initial state  $\rho_0$ , *the results obtained by measuring  $C$  depend on whether or not  $A$  was measured*, even though  $A$  is spacelike to  $C$ . By arranging beforehand that  $B$  will certainly be measured, someone at  $O_1$  could clearly use this dependence of  $C$  on  $A$  to transmit information “superluminally” to a friend at  $O_3$ .<sup>1</sup>

#### A Simple Example with Coupled Systems

To see this effect at its simplest it may help to retreat from quantum field theory to a more elementary situation, namely a pair of coupled quantum systems together with three observables:  $A$  belonging to the first subsystem,  $C$  belonging to the second, and  $B$  being a joint observable of the combined system. As in the field-theory case, the effect of measuring  $B$  will be to make a prior intervention on the first subsystem felt by the second.

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<sup>1</sup> This effect is reminiscent of the acausality of [1], but in that case, information is transmitted directly into the past, rather than over spacelike separations, as here. Also, the acausality there depends on the assumption that one can directly “observe” spacetime properties of a history which need not correspond to any traditionally defined operator in Hilbert space. Here, in contrast, even the traditionally defined observables give trouble.

To simplify the analysis further we can make a change which is actually a generalization. Instead of considering specifically a measurement at  $O_1$  [respectively, a measurement on the first subsystem] we consider an arbitrary intervention, implemented mathematically by a unitary operator  $U$  formed (like  $A$  itself) from the restriction of  $\Phi$  to  $O_1$  [respectively a unitary element of the observable algebra of the first subsystem]. That a measurement is effectively a special case of this kind of intervention follows from the observation that the effect of a measurement of the observable  $A$  on the density-operator  $\rho$  is to convert it into the  $\lambda$ -average of  $\exp(-i\lambda A)\rho\exp(i\lambda A)$ , as a one-line calculation will confirm. (Here, of course, I mean the effect of the measurement before the value of the result is taken into account.) Thus, measuring  $A$  is equivalent to applying  $U = \exp(-i\lambda A)$  with a random value of the parameter  $\lambda$ , and in this sense is a special kind of “unitary intervention”. It follows in particular that if unitary intervention cannot transmit information, then neither can any measurement.

Incidentally, the subsuming of measurement under unitary intervention in this way leads to a more unified criterion for a theory to be “local”: it is local if interventions confined to some region can affect only the future of that region. But it is interesting that, although it no longer mentions measurement directly, this criterion is still expressed in terms of intervention by an external agent. As far as I know there is no way to directly express the idea of locality in the context of a completely self-contained system.

A specific example of the non-local influence in question is now easy to obtain. Let the two subsystems be spin-1/2 objects, and let their initial state be  $|dd\rangle$ , where the two spin-states are  $u$  and  $d$ . At time  $t_1$  let us “kick” the first subsystem (or more politely “intervene”) by exchanging  $u$  with  $d$  (we apply the unitary operator  $\sigma_1$ ). At time  $t_2$  we measure the operator  $B$  of orthogonal projection onto the state  $(|uu\rangle + |dd\rangle)/\sqrt{2}$ ; and at  $t_3$  we measure an arbitrary observable  $C$  of the second subsystem.

It is straightforward to work out the density-operator which governs this final measurement of  $C$  by “tracing out” the first subsystem from the state resulting from the  $B$ -measurement. The result is that the second subsystem ends up in the pure spin-down state,  $|d\rangle\langle d|$ , and the expectation-value of  $C$  is accordingly  $\langle d|C|d\rangle$ . In contrast, without the kick, we would have obtained an entirely different effective state, namely the totally random density operator  $(1/2)(|u\rangle\langle u| + |d\rangle\langle d|)$ , corresponding to a  $C$ -expectation of  $(1/2)\text{tr}C$ . Thus, the detection of spin-up for the second subsystem would be an unambiguous signal that the first subsystem had not been kicked.

If you feel uneasy about using a kick rather than an actual measurement, you can replace the intervention at  $O_1$  with a measurement of  $A = \sigma_1$  (we apply  $\exp(i\sigma_1\lambda)$  with

a random value of  $\lambda$  instead of with  $\lambda = \pi/2$ ). The computation is not much longer, and yields for the effective state of subsystem 2, the density-operator  $(1/4)|u\rangle\langle u| + (3/4)|d\rangle\langle d|$ . This again differs from the totally random state, though not so strikingly as before.

### An Example in Quantum Field Theory

In a sense, the two subsystem example just given is all we need, since one would expect to be able to embed it in any quantum field theory which is sufficiently general to be realistic. Still, one might worry about non-localities having snuck in in connection with the particle concept on which the identification of the subsystems would probably be based in such an embedding; so it seems best to present an example couched directly in terms of a quantum field and its observables.

The example will follow the lines of that just given, but the computation is a bit more involved, and I will present it in slightly more detail, working for convenience in the interaction picture, for which the field  $\phi$  evolves independently of the intervention, while the state  $\rho$  get “kicked” from its initial value  $\rho_0$  to  $U\rho_0U^*$ ,  $U$  being the unitary operator which implements the kick. The quantum field will be a free scalar field  $\phi(x)$  initially in its vacuum state, and the three spacetime regions will be those introduced above. The kicking operator will be taken to be  $U = \exp(i\lambda\phi(y))$ , where  $y \in O_1$ ; and the observable measured in  $O_3$  will be  $C = \phi(x)$ , where  $x \in O_3$ . (Really the fields should be smeared, but it will be clear that this would make no difference.) Finally, the observable  $B$  measured in  $O_2$  (which can be chosen as any operator at all, since  $O_2$  includes the whole spacetime in its domain of dependence) will be orthogonal projection onto the state-vector

$$|b\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{2}$$

where  $|0\rangle$  is the vacuum and  $|1\rangle$  is some convenient one-particle state; thus  $B = |b\rangle\langle b|$ .

Denoting vacuum expectation values simply by  $\langle \cdot \rangle$ , we may express the mean value predicted for  $C$  as

$$\langle U^*BCBU \rangle + \langle U^*(1-B)C(1-B)U \rangle, \tag{3}$$

whose two terms correspond respectively to the outcomes 1 and 0 for the  $B$ -measurement. [To derive this expression, we may begin with the state  $\rho = U\rho_0U^*$ , as it is after the kick but before the measurement of the projection  $B$ . The probability of the outcome  $B = 1$  is then  $\text{tr}\rho B = \text{tr}\rho_0U^*BU = \langle U^*BU \rangle$ , and that of the outcome  $B = 0$  is  $\langle U^*(1-B)U \rangle$ . In the former case, the projection postulate yields for the consequent (normalized) state,

$$\sigma = \frac{B\rho B}{\text{tr}B\rho B} = \frac{B\rho B}{\text{tr}\rho B},$$

and therefore for the consequent expectation value of  $C$ ,

$$\text{Exp}(C|B=1) = \text{tr} \sigma C = \frac{\text{tr} \rho B C B}{\text{tr} \rho B}.$$

When weighted with the probability  $\text{tr} \rho B$  of actually obtaining  $B=1$ , this expression becomes the contribution of the  $B=1$  outcome to the final expectation-value of  $C$ :

$$\text{Exp}(C, B=1) = \text{tr} \rho B C B = \langle U^* B C B U \rangle.$$

Finally, adding in the contribution of the  $B=0$  outcome yields (3), as desired.]

In order that the mean value (3) not depend on the magnitude of the kick, it is necessary in particular that its derivative with respect to  $\lambda$  vanish at  $\lambda=0$  (or in other words that an infinitesimal kick have no effect). Now, this derivative may easily be computed, and turns out to be (twice) the imaginary part of

$$\langle \phi(y)(C + 2BCB - BC - CB) \rangle,$$

which therefore must be purely real in order that locality be respected. However the first and last terms are separately real; the former equals  $\langle \phi(y)\phi(x) \rangle$ , which is real because  $x \not\# y$  (a notation meaning that  $x$  is spacelike to  $y$ ) whence  $\phi(x) \not\# \phi(y)$  (a notation meaning that  $\phi(x)$  and  $\phi(y)$  commute), and the latter reduces to  $|\alpha|^2$  times the former when the definition of  $B$  is used. With the aid of the notation,  $\psi(x) \equiv \langle 0|\phi(x)|1 \rangle$ , the results of combining the two remaining terms can be written as

$$2(\alpha^* \beta)^2 \psi(x)\psi(y) + (2|\alpha|^2 - 1)|\beta|^2 \psi(x)^* \psi(y). \quad (4)$$

Here the star denotes complex conjugation, and the fact that  $\phi(x)$  changes the particle number by  $\pm 1$  has been used in places to eliminate some terms which would have been present had  $\phi(x)$  not been a free field.

To show that (4) need not be real, we can, for example, eliminate its second term by taking  $|\alpha|^2 = |\beta|^2 = 1/2$ . What remains can then be given any desired phase by an appropriate choice of the relative phase of  $\alpha$  and  $\beta$ , unless it happens that  $\psi(x)$  or  $\psi(y)$  vanishes. Avoiding this possibility in our choice of  $\psi$  (indeed it would be difficult not to avoid it, since  $\psi$  is purely positive frequency!), we arrive at the conclusion announced earlier. Notice, incidentally, that we could have arranged  $\psi(\cdot)$  to manufacture a problem even with  $\alpha=0$ , but the superposition with the vacuum state allows us to control the phase of (4) for an arbitrary choice of the one-particle wave-function  $\psi$ . On the other hand, setting  $\alpha=0$  in (2) does have the advantage of lending a particularly simple physical meaning to the measurement  $B$ : it merely asks whether or not there is precisely one particle present and if so whether that particle is in the specific state,  $|1\rangle$ .

#### 4. POSSIBLE IMPLICATIONS FOR QUANTUM FIELD THEORY AND QUANTUM GRAVITY

In a way it is no surprise that a measurement such as of  $B$ , which occupies an entire hypersurface, should entail a physical non-locality; but surprising or not, the implications seem far from trivial. Unless one admits the possibility of superluminal signaling, the entire interpretive framework constructed above for the quantum field formalism must be rejected as it stands. What then remains of the apparatus of states and observables, on which the interpretation of quantum mechanics is traditionally based?

A possible way to salvage our framework would be to further restrict the allowed measurement-regions  $O_j$  in such a manner that the transitive closure we took in defining  $\prec$  would be redundant. For example, we could require that for each pair of regions  $O_j, O_k$ , all pairs of points  $x \in O_j$  and  $y \in O_k$  be related in the same way (i.e. either  $x \not\parallel y$  in all cases, or  $x < y$  in all cases, or  $x > y$ ). Such a restriction would block the kind of example just presented, but it would also be a very severe limitation on the allowed measurements (excluding measurements on Cauchy surfaces, for example.) Also it is difficult to see how the ability to perform a measurement in a given region—or the effect of that measurement on future probabilities—could be sensitive to whether some other measurement was located totally to its past, or only partly to its past and partly spacelike to it.<sup>2</sup>

Another way out might be to select the allowed measurements on some more ad hoc basis than that which was set up in Section 2. For example it can be shown, in the situation with the coupled subsystems, that information transfer never occurs when  $B$  is a *sum* of observables, one belonging to each subsystem. This suggests that one might allow Cauchy-surface observables which were integrals of local operators, even though other Cauchy-surface observables would still have to be excluded. Spatially smeared fields have this additive character, for example (though they might be very singular as operators), but fields which are also smeared in time do not. Similarly, the most obvious gauge-invariant hypersurface observables in nonabelian gauge theories like QCD, i.e. holonomies, are functions of the commuting set of local variables  $A_j(x)$ , but they are not linear functions. This puts them in jeopardy because, in the two subsystem situation, nonlinear combinations of pairs of observables do in general lead

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<sup>2</sup> On the other hand, there exists a purely formal consideration which suggests that in fact there might be some difference between the two cases. If one demands an answer to “where does the collapse occur?”, the only viable response would seem to be “along the past light cone”, and that would indeed appear as an influence of a later measurement on an earlier one, when the latter is partly spacelike to the former.

to information transfer. (For example, even something as simple as the product of two projections has this difficulty).<sup>3</sup> This also portends trouble for diffeomorphism-invariant hypersurface observables in quantum gravity.

This may be an appropriate place to comment on one of the few attempts I know of to design concrete models of field measurements, namely that of Bohr and Rosenfeld [2]. Their idealized apparatus is designed to measure averaged field values in an arbitrary pair of spacetime regions (even overlapping ones!). When their regions are related as we required in Section 2, their results are consistent with the conclusion that their procedure does indeed furnish ideal measurements, as we defined this term earlier. Specifically, the apparatus interacts with the field only in the two specified regions, and the uncontrollable disturbance exerted by the earlier measurement on the later one is no greater than required by the commutator of the corresponding operators. It would thus seem important to extend (or reinterpret) their essentially classical treatment of the apparatus to a quantum one, in order to learn how close they come to actually fulfilling the requirements for an ideal measurement. Specifically, one can ask whether they actually measure the field averages they claim to, and whether the probabilities of the different possible outcomes are those predicted by the quantum formalism (with special reference to the use of the projection postulate after the first measurement, since its effect could *only* be seen in a full quantum treatment). Such an analysis would be especially interesting (even though Bohr and Rosenfeld only treat a free field) because there is no obvious formal reason why their temporally smeared observables should not suffer from the type of non-locality we have been discussing here.

However such an analysis would turn out, though (and it doesn't look that hard to do), there remains the more general fact that the need to resort to a case-by-case analysis would still leave us without any clear formal criterion for which "observables" can be ideally-measured, and which cannot; and we might also be left without any general rule to take the place of the projection postulate. Moreover the charm of the "transformation theory" would be lost as well, since different orthonormal bases would no longer be equal before the Law of Locality (in the case of hypersurface measurements, for example).

Now, as we are all aware, the question of what is the best dynamical framework for quantum gravity is not one which everyone will answer in the same way. It is very possible that some as yet unknown framework will be needed, but among existing

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<sup>3</sup> An interesting problem would be to characterize which joint observables of a pair of subsystems potentially lead to "information transfer", and which don't.

interpretations of quantum mechanics, I have long felt that the sum-over-histories is the most promising, both for philosophical reasons and for practical ones. It has the great advantage that it deals with spacetime rather than just space, so that what is usually called the “problem of time” hardly makes an appearance. In particular, notions like horizon-area are well-defined, and quantum cosmology can investigate the early history of the universe because the universe really does have a history.<sup>4</sup>

In the non-relativistic context, however, the sum-over-histories has the disadvantage of allowing you to ask “too many” questions, including ones whose answering seems to lead to causality violations similar to those of Section 3 [1]. And in the face of such difficulties, one lacks a well-defined criterion to know in advance what measurements are possible, or even what interactions should count as a measurement. In these ways, however, the sum-over-histories would now appear to be no worse off than what survives of the state-and-observable framework when one tries to extend it to flat space quantum field theory (not to mention quantum gravity!)<sup>5</sup>

With the formal notion of measurement compromised as it seems to be already in quantum field theory, the greatest advantage of the sum-over-histories may be that it does not employ measurement as a basic concept. Instead it operates with the idea of a *partition* (or “coarse-graining”) of the set of all histories, and assigns probabilities directly to the members of a given partition, using what I would call the quantum replacement for the classical probability calculus.

Actually, there are (at least) two variants of this idea. In the way I like to think about it [3], the partition is “implemented” by a designated subsystem which gains information about the rest of the universe. In this way of thinking, the basic idea would be that of probability *relative to* such a subsystem, and the difficulty is that it is not fully clear under what conditions information is actually obtained. (In practice, though, no problem is apparent in either of the two extreme cases which the history of science has brought us so far — laboratory experiments and astronomical

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<sup>4</sup> Another approach which deserves mention here is that of David Finkelstein, who views dynamics in terms of networks of elementary processes of input/output or creation/annihilation, and correspondingly draws a fundamental distinction between preparations and detections. Clearly this is relevant to the examples of Section 2, since our reasoning there required an ideal measurement to be both a detection and a preparation at once.

<sup>5</sup> The sum-over-histories also suffers esthetically from its inability to incorporate the unitary symmetry of the so called transformation theory. But we have seen that the Hilbert-space framework is now no better off in this respect either.

observations.) In the other approach [4], the partition is given a priori, but subjected to a condition of decoherence. In this approach, the classical probability calculus applies, and one effectively returns to a classical (but stochastic) dynamical law. Some drawbacks of this variant are that one must do a very difficult analysis even to authorize the use of probability, and (more fundamentally) that decoherence is always *provisional*, since it applies to entire histories and is therefore always in danger of being overturned by future activities.

Let us hope that future activities by all of us will clarify some of these issues, and bring out the new insights which further study of the “quantum measurement problem” undoubtedly has to offer. After all, the dialectical inter-penetration of all existing things is at the heart of the interpretation which Bohr wanted to give to the new quantum formalism. In the textbook formulation this inseparability shows up in the impossibility of observing without also disturbing. It will be interesting to see how it shows up in the modified dynamical framework which will have to be developed for quantum gravity, and, as we see now, perhaps even for quantum field theory.

In concluding I would like to dedicate this article to Dieter Brill, in honor of his sixtieth birthday and in recollection of the many happy hours I have spent discussing physics with him (and once in a while playing sonatas together). Happy Birthday, Dieter!

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## REFERENCES

- [1] Sorkin, R.D., “Problems with Causality in the Sum-over-histories Framework for Quantum Mechanics”, in A. Ashtekar and J. Stachel (eds.), *Conceptual Problems of Quantum Gravity* (Proceedings of the conference of the same name, held Osgood Hill, Mass., May 1988), 217–227 (Boston, Birkhäuser, 1991).
- [2] Bohr, N. and L. Rosenfeld, “Zur Frage der Messbarkeit der Elektromagnetischen Feldgrößen”, *Det Kgl. Danske Videnskabernes Selskab., Matematisk-fysiske Meddelelser*, **12**, No. 8 (1933); “Field and Charge Measurements in Quantum Electrodynamics”, *Phys. Rev.* **78**:794-798 (1950).
- [3] Sorkin, R.D., “On the Role of Time in the Sum-over-histories Framework for Gravity”, paper presented to the conference on The History of Modern Gauge Theories,

held Logan, Utah, July, 1987, to be published in *Int. J. Theor. Phys.* (1994, to appear); Sinha, Sukanya and R.D. Sorkin, “A Sum-Over-Histories-Account of an EPR(B) Experiment” *Found. of Phys. Lett.*, **4**, 303-335, (1991).

- [4] For a review see: Hartle, J.B., “The Quantum Mechanics of Cosmology”, in *Quantum Cosmology and Baby Universes: Proceedings of the 1989 Jerusalem Winter School for Theoretical Physics*, eds. S. Coleman et al. (World Scientific, Singapore, 1991)