

Using the $\omega\pi^-$ Mass Spectrum in $B \rightarrow D^{(*)}\omega\pi^-$ Decays To Determine the Mass and Width of the ρ'

Abstract

We report on a new more sophisticated method for fitting the $\omega\pi^-$ invariant mass spectrum in $\bar{B} \rightarrow D^{(*)}\omega\pi^-$ decays. We find the mass and width of ρ' to be $(1336 \pm 24 \pm 9)$ MeV and $(510 \pm 58 \pm 61)$ MeV, respectively.

1 Introduction

Previously, we had shown the first evidence for B in $D^*\omega\pi^-$ and $D\omega\pi^-$ modes, where the $\omega\pi^-$ formed a wide resonance that we identified as the elusive ρ' [1]. The mass distribution shown in Fig. 1 is fit to a classic Breit-Wigner function of the form:

$$BW = \frac{\Gamma}{(M_{\omega\pi} - M_{\rho'})^2 + \Gamma^2/4} \quad (1)$$

Where $M_{\omega\pi}$ is the varying $\omega\pi$ invariant mass, and $M_{\rho'}$ and Γ are the nominal mass and width of the ρ' resonance. The mass and width resulting from this fit are (1418 ± 26) MeV and (382 ± 44) MeV, respectively.

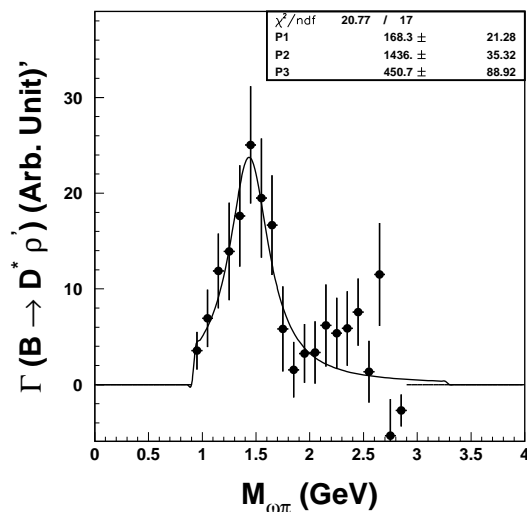


Figure 1: Classic Breit-Wigner fit on the distribution of $M_{\omega\pi}$ in $B \rightarrow D^*\omega\pi^-$ mode.

Since the ρ' decays to $\omega\pi^-$ via a P-wave, we need to take into account the fact that the width, $\Gamma(M_{\omega\pi})$, may not be constant, but can vary with $M_{\omega\pi}$. Furthermore we need to consider the kinematic limits from $B \rightarrow D^{(*)}\rho'$ decay and $\rho' \rightarrow \omega\pi$ decay.

Alan Weinstein's CBX note on Breit-Wigner functions [2] and further communications from Alan have been crucial.

2 The Differential Decay Distribution

We can write a general expression for the the differential distribution for $M_{\omega\pi}$ in $d\Gamma(B \rightarrow D^*\omega\pi)$ via the ρ' , following Weinstein's work as:

$$d\Gamma(B \rightarrow D^*\omega\pi) = \frac{1}{2M_B} |A(B \rightarrow D^*\rho') BW(\rho') A(\rho' \rightarrow \omega\pi)|^2 \times d\mathcal{P}(B \rightarrow D^*\rho') d\mathcal{P}(\rho' \rightarrow \omega\pi) \frac{dM_{\omega\pi}^2}{2\pi}, \quad (2)$$

where $d\mathcal{P}$ indicates a phase space term, BW indicates some form of a Breit-Wigner shape function, and A indicates an amplitude.

The phase space of $B \rightarrow D^*\rho'$ provides a cut off at higher $M_{\omega\pi}$. Whereas the phase space of $\rho' \rightarrow \omega\pi$ provides a cut off at lower $M_{\omega\pi}$. We assume that the two decay stages are independent and can be factorized. Thus the function can be calculated from the widths of B and ρ' decays.

2.1 The B Decay Width

The width for ρ' production is given by:

$$\Gamma(B \rightarrow D^*\rho') = \frac{1}{2M_B} |A(B \rightarrow D^*\rho')|^2 d\mathcal{P}(B \rightarrow D^*\rho') \quad (3)$$

$$d\mathcal{P}(B \rightarrow D^*\rho') = \frac{1}{8\pi} \left(\frac{2p_{D^*}}{M_B} \right) \quad (4)$$

$$A(B \rightarrow D^*\rho') \sim \sqrt{G_F} V_{bc} \times \text{Lorentz structure} \times g(M_{\omega\pi}) \quad (5)$$

where $g(M_{\omega\pi})$ is the ρ' weak production form factor.

Instead of having to calculate $g(M_{\omega\pi})$, we can use our knowledge of semileptonic b decays, $B \rightarrow D^{(*)}\ell^-\bar{\nu}$ coupled with factorization to approximate $A(B \rightarrow D^*\rho')$.

Factorization tell us:

$$\Gamma(B \rightarrow D^*\rho') = 6\pi^2 |V_{ud}|^2 f_{\rho'}^2 |a_1|^2 \frac{d\Gamma(B \rightarrow D^*\ell\nu)}{dq^2} \Big|_{q^2=M_{\omega\pi}^2} \propto f_{\rho'}^2 p_{D^*} M_{\omega\pi}^2 \times \{|H_+(M_{\omega\pi}^2)|^2 + |H_-(M_{\omega\pi}^2)|^2 + |H_o(M_{\omega\pi}^2)|^2\} \quad (6)$$

The helicity amplitudes $H(M_{\omega\pi}^2)$ can be related to the axial-vector form factors $A_1(q^2)$ and

$A_2(q^2)$, and vector form factor $V(q^2)$. Details can be found in reference [3].

$$\begin{aligned}
H_{\pm}(q^2) &= (M_B + M_{D^*}) A_1(q^2) \mp \frac{2M_B p_{D^*}}{M_B + M_{D^*}} V(q^2) \\
H_o(q^2) &= \frac{1}{2M_{D^*}\sqrt{q^2}} \left[(M_B^2 - M_{D^*} - q^2)(M_B + M_{D^*}) A_1(q^2) - \frac{4M_B^2 p_{D^*}^2}{M_B + M_{D^*}} A_2(q^2) \right] \quad (7)
\end{aligned}$$

In the heavy-quark symmetry limit, the form factors A_1, A_2 , and V are related to the Isgur-Wise function. With correction due to finite heavy quark mass and α_s , they can be written as

$$\begin{aligned}
A_1(q^2) &= \left[1 - \frac{q^2}{(M_B + M_{D^*})^2} \right] \frac{M_B + M_{D^*}}{2\sqrt{M_B M_{D^*}}} h_{A_1}(w) \\
A_2(q^2) &= R_2 \frac{M_B + M_{D^*}}{2\sqrt{M_B M_{D^*}}} h_{A_1}(w) \\
V(q^2) &= R_1 \frac{M_B + M_{D^*}}{2\sqrt{M_B M_{D^*}}} h_{A_1}(w) , \quad (8)
\end{aligned}$$

where w is the invariant four-velocity transfer.

The values calculated by Neubert for R_1 and R_2 have the explicit dependence on w [4] of

$$\begin{aligned}
R_1(w) &= 1.35 - 0.22(w - 1) + 0.09(w - 1)^2 \\
R_2(w) &= 0.79 + 0.15(w - 1) - 0.04(w - 1)^2 , \quad (9)
\end{aligned}$$

while Close and Wambach [5] determine

$$\begin{aligned}
R_1(w) &= 1.15 - 0.07(w - 1) + \mathcal{O}(w - 1)^2 \\
R_2(w) &= 0.91 + 0.04(w - 1) + \mathcal{O}(w - 1)^2 . \quad (10)
\end{aligned}$$

CLEO measured $R_1(0)$ and $R_2(0)$ to be $1.18 \pm 0.30 \pm 0.12$ and $0.71 \pm 0.22 \pm 0.07$, respectively [6]. The form factor $h_{A_1}(w)$ can be assumed to be linear as $h_{A_1}(w) = 1 - \rho_{A_1}^2(w - 1)$; CLEO measured $\rho_{A_1}^2$ to be $0.91 \pm 0.15 \pm 0.06$.

$\Gamma(B \rightarrow D^* \rho')$ as function of $M_{\omega\pi}$ is shown in Fig. 2, where the form factor $f_{\rho'}$ is not included.

Now for the ρ' decay form-factor, $f_{\rho'}$ in Eq. 6, we use the ISGW prediction.

$$f_{\rho'}(q^2) = f_{\rho'}(q_{max}^2) \times \exp\{-(0.03 \text{ GeV}^{-2})(q_{max}^2 - q^2)\} \quad (11)$$

2.2 The Width of ρ' decay

The width of ρ' decay can be expanded as:

$$\Gamma(\rho' \rightarrow \omega\pi) = \frac{1}{2M_{\omega\pi}} |A(\rho' \rightarrow \omega\pi)|^2 d\mathcal{P}(\rho' \rightarrow \omega\pi) \quad (12)$$

$$d\mathcal{P}(\rho' \rightarrow \omega\pi) = \frac{1}{8\pi} \left(\frac{2p_{\omega}}{M_{\omega\pi}} \right) \quad (13)$$

$$A(\rho' \rightarrow \omega\pi) = \text{Lorentz structure} \times h(M_{\omega\pi}^2) , \quad (14)$$

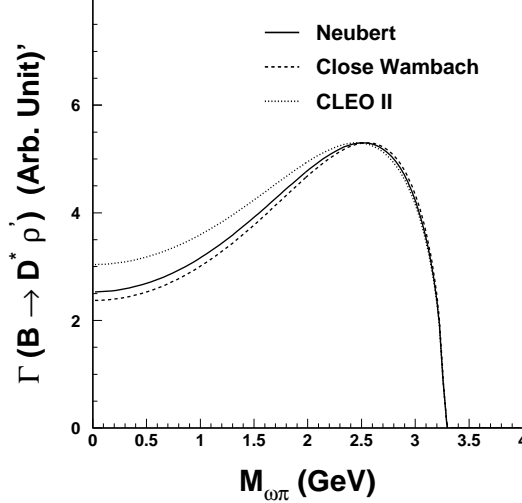


Figure 2: Decay width of $B \rightarrow D^* \rho'$ as function of $M_{\omega\pi}$. The parameters R_1 and R_2 are from Neubert (solid line), Close and Wambach (dashed line) and the CLEO measurement (dotted line), with $\rho_{A_1}^2$ as 0.91.

where p_ω is evaluated in the $\omega\pi$ rest frame, and $h(M_{\omega\pi}^2)$ is the ρ' strong decay form factor, which is usually assumed to be a constant.

The Lorentz structure of $\rho' \rightarrow \omega\pi$ must be linear in the polarization vectors of the ρ' and the ω . The simplest mathematical expression is $\varepsilon_{\rho'} \cdot \varepsilon_\omega$, which is S-wave. For P-wave, since $\varepsilon_{\rho'} \cdot p_{\rho'} = 0$ and $\varepsilon_\omega \cdot p_\omega = 0$ by transversality, the only Lorentz scalars we can form are $(\varepsilon_{\rho'} \cdot p_\omega)(\varepsilon_\omega \cdot p_{\rho'})$ and $\epsilon_{\mu\nu\alpha\beta} \varepsilon_{\rho'}^\mu \varepsilon_\omega^\nu p_\omega^\alpha p_{\rho'}^\beta$. The first term violates parity conservation. In appendix, we describe the detailed calculation of the second term, which gives

$$|A(\rho' \rightarrow \omega\pi)|^2 = h^2(M_{\omega\pi}^2) |\epsilon_{\mu\nu\alpha\beta} \varepsilon_{\rho'}^\mu \varepsilon_\omega^\nu p_\omega^\alpha p_{\rho'}^\beta|^2 \propto h^2(M_{\omega\pi}^2) M_{\omega\pi}^2 p_\omega^2 \quad (15)$$

Thus, the width of $\rho' \rightarrow \omega\pi$ is:

$$\Gamma(\rho' \rightarrow \omega\pi) \propto h^2(M_{\omega\pi}^2) p_\omega^3 \quad (16)$$

Note, that the contribution of longitudinal polarized ω to the width is zero. This was shown in our previous spin-parity study [1]. Our measurements gave $\Gamma_{Long}/\Gamma_{tot}$ to be $10 \pm 9\%$ in $D^* \omega\pi$ mode and $-0.4 \pm 22\%$ in $D\omega\pi$ mode.

2.3 The Breit-Wigner

The Breit-Wigner function has a long history. Fundamentally we are approximating the decay as having a non-changing amplitude as a function of mass in the simplest case. In general the denominator of the Breit-Wigner has a fixed form in amplitude given by

$$\text{Denominator} = M_{\omega\pi}^2 - M_{\rho'}^2 - iM_{\omega\pi}\Gamma_{tot}(M_{\omega\pi}), \quad (17)$$

The numerator is where we have to have more discussion. Since $\omega\pi$ is a large decay mode of ρ' , we assume that the mass dependence of Γ_{tot} can be approximated by the mass

dependence of $\Gamma_{\omega\pi}$

$$\Gamma_{tot}(M_{\omega\pi}) = \Gamma_{tot}(M_{\rho'}) \frac{\Gamma_{\omega\pi}(M_{\omega\pi})}{\Gamma_{\omega\pi}(M_{\rho'})} \quad (18)$$

From the derivation of $\Gamma(\rho' \rightarrow \omega\pi)$ above, we see that

$$\frac{\Gamma_{\omega\pi}(M_{\omega\pi})}{\Gamma_{\omega\pi}(M_{\rho'})} = \left(\frac{h(M_{\omega\pi}^2)}{h(M_{\rho'})} \right)^2 \times \left(\frac{p_{\omega}(M_{\omega\pi})}{p_{\omega}(M_{\rho'})} \right)^3 \quad (19)$$

2.4 The Differential Distribution

Eq. 2 now can be rewritten as:

$$d\Gamma(B \rightarrow D^* \omega\pi) = \Gamma(B \rightarrow D^* \rho') \frac{2M_{\omega\pi} \Gamma_{\omega\pi}(M_{\omega\pi})}{(M_{\omega\pi}^2 - M_{\rho'}^2)^2 + M_{\omega\pi}^2 \Gamma_{tot}(M_{\omega\pi})^2} \frac{dM_{\omega\pi}^2}{2\pi} \quad (20)$$

With the relation shown in Eq. 18, we have the differential function which can be used for fitting the mass distribution:

$$\frac{d\Gamma(B \rightarrow D^* \omega\pi)}{dM_{\omega\pi}} = C \times \Gamma(B \rightarrow D^* \rho') \times \frac{\Gamma M_{\omega\pi}^2}{(M_{\omega\pi}^2 - M_{\rho'}^2)^2 + M_{\omega\pi}^2 \Gamma^2}, \quad (21)$$

where Γ is the $M_{\omega\pi}$ -dependent total width. From Eq. 18 and Eq. 19 we get that:

$$\Gamma = \Gamma_0 \times \left(\frac{h(M_{\omega\pi}^2)}{h(M_{\rho'})} \right)^2 \times \left(\frac{p_{\omega}(M_{\omega\pi})}{p_{\omega}(M_{\rho'})} \right)^3 \quad (22)$$

2.5 Weak Decay Form Factor of $\rho' \rightarrow \omega\pi$

Now, what form should we use for $h(M_{\omega\pi}^2)$? In Eq. 15, the dimensions of $h(M_{\omega\pi}^2)$ are GeV^{-1} . But the dependence on $M_{\omega\pi}$ could be anything. So we can try $h(M_{\omega\pi}^2) \propto M_{\omega\pi}^n$, where n is allowed to float in the fit.

$h(M_{\omega\pi}^2)$ could also include Blatt-Weisskopf factors [7]. A Blatt-Weisskopf factor for the P-wave decay $\rho' \rightarrow \omega\pi$ would be of the form:

$$FF(M_{\omega\pi}^2) = \frac{1 + (Rp_{\omega}(M_{\omega\pi}))^2}{1 + (Rp_{\omega}(M_{\rho'}))^2} \quad (23)$$

where R is the radius of the ρ' meson. A typical value is $R = 1 \text{ fm}/\hbar c$.

3 Fit to $M_{\omega\pi}$ Spectrum

Since we are limited by the statistics, we chose to add the D and D^* final states together. This is permissible since the Lorentz structure of ρ' decay in the different modes is the same. The only difference is in the width of $B \rightarrow D^{(*)} \rho'$ part, that depends on $M_{\omega\pi}$, however this difference is slight.

We use following formula to represent the width:

$$\Gamma(M_{\omega\pi}) = \Gamma(M_{\rho'}) \left(\frac{p_{\omega}(M_{\omega\pi})}{p_{\omega}(M_{\rho'})} \right)^3 \left(\frac{M_{\rho'}}{M_{\omega\pi}} \right)^n \frac{1 + (Rp_{\omega}(M_{\omega\pi}))^2}{1 + (Rp_{\omega}(M_{\rho'}))^2} \quad (24)$$

We allow the parameters R and n to float. We use Neubert's calculated values for R_1 and R_2 ; effects of changing this to other estimates will contribute to the systematic error. Fig. 3 shows the fit to $M_{\omega\pi}$ distribution.

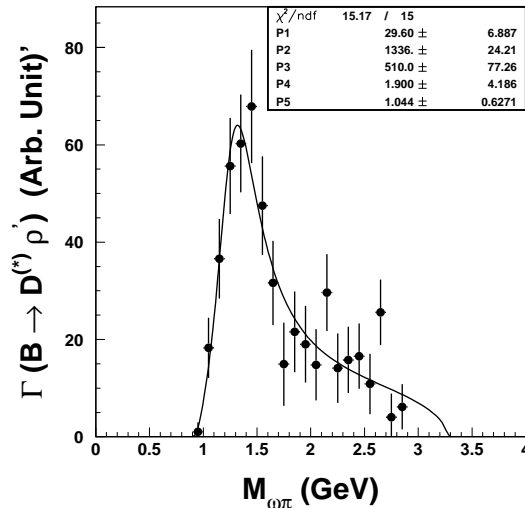


Figure 3: Fit to $M_{\omega\pi}$ distribution, where R_1 and R_2 are from Neubert calculation.

The fit gives the mass and width to be (1336 ± 24) MeV and (510 ± 58) MeV, respectively. The values for R and n are $1.90^{+7.35}_{-1.03}$ and $1.04^{+0.65}_{-0.61}$, respectively.

We estimate the systematic errors from these sources:

- We change the value of R and n by one standard deviation;
- In the fit, we use D^* mass as the mass of $D^{(*)}$. We change to the D mass.
- Instead of using R_1 and R_2 from Neubert's calculation, we use the calculation from Close and Wambach or the CLEO measurement of these parameters.

Listed in the Table 1 are the contributions from each sources. Thus our final value for the mass of ρ' resonance is $1336 \pm 24 \pm 9$ MeV. The width is $510 \pm 58 \pm 61$ MeV.

It is possible that values of the ρ' mass and width could be affected if additional resonance substructure of unknown origin were to be included. There is no evidence, however, that such structures are needed. By including all the known physics effects of phase space and Lorentz structure we are able to describe the data quite well.

4 Conclusions

We used more sophisticated form to fit the $M_{\omega\pi}$ spectrum from $B \rightarrow D^{(*)}\omega\pi$ decays. The mass and width of ρ' resonance is fitted to be $(1336 \pm 24 \pm 9)$ MeV, and $(510 \pm 58 \pm 61)$ MeV, respectively.

Table 1: Mass and width fit of resonance ρ'

Parameters	Mass (MeV)	Width (MeV)
Neubert	1336 ± 24	510 ± 58
$R \pm 1\sigma$	± 4.5	± 32
$n \pm 1\sigma$	± 1.0	± 49
$M = M_D$	-4.8	-10
Close-Wambach	-0.7	$+2.3$
CLEO $D^*l\nu$	$+6.4$	$+15$
Systematic error	± 9	± 61

5 Acknowledgements

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References

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6 Appendix: Calculation of the Lorentz Structure

In this section, we will calculate the Lorentz structure $|\epsilon_{\mu\nu\alpha\beta} \epsilon_{\rho'}^\mu \epsilon_\omega^\nu p^\alpha q^\beta|^2$, where we use p^α and q^β to represent the four momentum of ρ' and ω . In general the helicity structure is more apparent when calculated in a rest frame where the ρ' is in motion. Later we will give the expression in ρ' rest frame.

The transversality $\varepsilon_\omega \cdot p_\omega = 0$ [8]. For spin 1 massive particle with momentum $\vec{p} = p\vec{e}_z$, the helicity states can be represented as:

$$\varepsilon_{(\pm)} = \mp(0, 1, \pm i, 0) \quad (25)$$

$$\varepsilon_{(0)} = (p, 0, 0, E)/M \quad (26)$$

For general \vec{p} , we can construct $\varepsilon_{(0)}$ as follows. The $\varepsilon_{(\pm)}$ are not shown as there will not be directly used.

$$\varepsilon_{(0)} = \left(\frac{p}{M}, \frac{E \vec{p}}{M p} \right) \quad (27)$$

Now we need to prepare calculations of several quantities. First is $\varepsilon_{(\lambda)}^\mu \varepsilon_{(\lambda)}^\nu$. The summation over all helicity states is given in [8]. For helicity 0, we use Eq: 27 and expand it. We have

$$\sum_{\lambda=0,\pm} \varepsilon_{(\lambda)}^\mu \varepsilon_{(\lambda)}^\nu = -g^{\mu\nu} + \frac{p_\mu p_\nu}{M^2} \quad (28)$$

$$\varepsilon_{(0)}^\mu \varepsilon_{(0)}^\nu = \frac{p^\mu p^\nu}{M^2} - \delta_{\mu 0} \delta_{\nu 0} + \frac{(p^\mu p^\nu)^*}{P^2} \quad (29)$$

$$\varepsilon_{(+)}^\mu \varepsilon_{(+)}^\nu + \varepsilon_{(-)}^\mu \varepsilon_{(-)}^\nu = -g^{\mu\nu} + \delta_{\mu 0} \delta_{\nu 0} - \frac{(p^\mu p^\nu)^*}{P^2} \quad (30)$$

In $(p^\mu p^\nu)^*$, only $\mu, \nu = 1, 2, 3$ has non-zero value. In the later calculations, we always assume that transverse helicity states ($\lambda = \pm 1$) have same probabilities.

For terms like $\varepsilon_{\alpha\beta\gamma\delta} p^\alpha p^\beta \dots$, exchanging α and β will introduce a negative sign. Thus the value is zero.

$$\varepsilon_{\alpha\beta\gamma\delta} p^\alpha p^\beta \times \text{anything} = 0 \quad (31)$$

The Lorentz structure will be calculated with summation of all helicity states and different polarizations.

$$A_{tot} = A_{00} + A_{01} + A_{10} + A_{11} = \varepsilon_{\alpha\beta\gamma\delta} \varepsilon_{\kappa\lambda\mu\nu} (\varepsilon^\alpha \varepsilon^\kappa p^\beta p^\lambda)_{\rho'} (\varepsilon^\gamma \varepsilon^\mu q^\delta p^\nu)_\omega \quad (32)$$

where A_{01} for longitudinal polarized ρ' and transverse polarized ω . Others are similar.

There is one term which often shows up; expanding we find:

$$\begin{aligned} \varepsilon_{0ijk} \varepsilon_{0imn} p_j p_m q_k q_n &= p_2^2 q_3^2 + p_3^2 q_2^2 - 2p_2 p_3 q_2 q_3 \quad (i = 1) \\ &+ p_1^2 q_3^2 + p_3^2 q_1^2 - 2p_1 p_3 q_1 q_3 \quad (i = 2) \\ &+ p_1^2 q_2^2 + p_2^2 q_1^2 - 2p_1 p_2 q_1 q_2 \quad (i = 3) \\ &+ (p_1^2 q_1^2 + p_2^2 q_2^2 + p_3^2 q_3^2) - (p_1^2 q_1^2 + p_2^2 q_2^2 + p_3^2 q_3^2) \\ &= (p_1^2 + p_2^2 + p_3^2)(q_1^2 + q_2^2 + q_3^2) - (p_1 q_1 + p_2 q_2 + p_3 q_3)^2 \\ &= p^2 q^2 - (\vec{p} \cdot \vec{q})^2 = p^2 q^2 (1 - \cos^2 \theta) \end{aligned} \quad (33)$$

Now the summation over all helicity states:

$$\begin{aligned}
A_{tot} &= \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\kappa\lambda\mu\nu} \left(-g^{\alpha\kappa} + \frac{p^\alpha p^\kappa}{M^2} \right) p^\beta p^\lambda \left(-g^{\gamma\mu} + \frac{q^\gamma q^\mu}{m^2} \right) q^\delta q^\nu \\
&= \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\kappa\lambda\mu\nu} g^{\alpha\kappa} g^{\gamma\mu} p^\beta p^\lambda q^\delta q^\nu \\
&= -2\epsilon_{0\beta\mu\delta} \epsilon_{0\lambda\mu\nu} p^\beta p^\lambda q^\delta q^\nu \quad (\alpha = 0 \text{ or } \kappa = 0) \\
&\quad + \epsilon_{\alpha 0\mu\delta} \epsilon_{\alpha 0\mu\nu} p^0 p^0 q^\delta q^\nu \quad (\beta = \lambda = 0) \\
&\quad + \epsilon_{\alpha\beta\nu 0} \epsilon_{\alpha\lambda\mu 0} p^\beta p^\lambda q^0 q^0 \quad (\delta = \nu = 0) \\
&\quad + 2\epsilon_{\alpha 0\mu\delta} \epsilon_{\alpha\lambda\mu 0} p^0 p^\lambda q^\delta q^0 \quad (\beta = \nu = 0 \text{ or } \lambda = \delta = 0) \\
&= -2p^2 q^2 (1 - \cos^2\theta) + 2E_{\rho'}^2 q^2 + 2E_\omega^2 p^2 - 4E_{\rho'} E_\omega p q \cos\theta
\end{aligned} \tag{34}$$

Lorentz structures for longitudinal polarized ρ' and ω :

$$\begin{aligned}
A_{00} &= \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\kappa\lambda\mu\nu} \left(\frac{p^\alpha p^\kappa}{M^2} - \delta_{\alpha 0} \delta_{\kappa 0} + \frac{(p^\alpha p^\kappa)^*}{p^2} \right) p^\beta p^\lambda \left(\frac{q^\gamma q^\mu}{m^2} - \delta_{\gamma 0} \delta_{\mu 0} + \frac{(q^\gamma q^\mu)^*}{q^2} \right) q^\delta q^\nu \\
&= \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\kappa\lambda\mu\nu} (p^\alpha p^\kappa)^* p^\beta p^\lambda (q^\gamma q^\mu)^* q^\delta q^\nu \frac{1}{p^2 q^2} \\
&= 0
\end{aligned} \tag{35}$$

In the first step, only one of nine terms survive; seven terms are eliminated due to Eq. 31. The one which has four δ 's results in two 0 subscripts in each ϵ . In the second step, Eq. 31 limits that β, λ, δ and ν are all zero, which also gives zero.

Lorentz structures for longitudinal polarized ρ' and transverse polarized ω :

$$\begin{aligned}
A_{01} &= \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\kappa\lambda\mu\nu} \left(\frac{p^\alpha p^\kappa}{M^2} - \delta_{\alpha 0} \delta_{\kappa 0} + \frac{(p^\alpha p^\kappa)^*}{p^2} \right) p^\beta p^\lambda \left(-g^{\gamma\mu} + \delta_{\gamma 0} \delta_{\mu 0} - \frac{(q^\gamma q^\mu)^*}{q^2} \right) q^\delta q^\nu \\
&= \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\kappa\lambda\mu\nu} \delta_{\alpha 0} \delta_{\kappa 0} g^{\gamma\mu} p^\beta p^\lambda q^\delta q^\nu - \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\kappa\lambda\mu\nu} g^{\gamma\mu} \frac{(p^\alpha p^\kappa)^*}{p^2} p^\beta p^\lambda q^\delta q^\nu \\
&= -\epsilon_{0\beta\mu\delta} \epsilon_{0\lambda\mu\nu} p^\beta p^\lambda q^\delta q^\nu + \epsilon_{\alpha 0\mu\delta} \epsilon_{\kappa 0\mu\nu} \frac{(p^\alpha p^\kappa)^*}{p^2} p^0 p^0 q^\delta q^\nu \\
&= \left(\frac{p^0 p^0}{p^2} - 1 \right) p^2 q^2 (1 - \cos^2\theta) \\
&= M_{\rho'}^2 q^2 (1 - \cos^2\theta)
\end{aligned} \tag{36}$$

Similarly, Lorentz structures for transverse polarized ρ' and longitudinal polarized ω gives:

$$A_{10} = m_\omega^2 p^2 (1 - \cos^2\theta) \tag{37}$$

Lorentz structures for transverse polarized ρ' and ω

$$\begin{aligned}
A_{11} &= A_{tot} - A_{00} - A_{01} - A_{10} \\
&= 2E_{\rho'}^2 q^2 + 2E_\omega^2 p^2 - 4E_{\rho'} E_\omega p q \cos\theta - (2p^2 q^2 + M_{\rho'}^2 q^2 + m_\omega^2 p^2) (1 - \cos^2\theta) \\
&= (E_{\rho'}^2 + E_\omega^2 p^2) (1 + \cos^2\theta) - 4E_{\rho'} E_\omega p q \cos\theta
\end{aligned} \tag{38}$$

In the rest frame of ρ' , the Lorentz structures are expressed:

$$\begin{aligned}A_{tot} &= 2M_{\rho'}^2 q_{\omega}^2 \\A_{00} &= 0 \\A_{01} &= M_{\rho'}^2 q_{\omega}^2 (1 - \cos^2 \theta) \\A_{10} &= 0 \\A_{11} &= M_{\rho'}^2 q_{\omega}^2 (1 + \cos^2 \theta)\end{aligned}\tag{39}$$

Please note that there is no contribution from longitudinal polarized ω .