Real Meanings to Complex Maxwell Fields

Ted Newman,
Dept of Physics and Astronomy, University of Pittsburgh,
Pgh., PA. 15260

July 29th 2003

(abstract)

We study and report on the physical meaning of an unusual class of Maxwell fields in Minkowski space and their likely generalization to a certain class of Einstein-Maxwell fields.

Though we are concerned with real Maxwell fields on real Minkowski space, it is easiest to discuss them from the following complex point of view. We first write the real Maxwell vectors

$$\overrightarrow{E} \;\;\&\;\; \overrightarrow{B}$$

as the complex vector

$$\overrightarrow{W} = \overrightarrow{E} + i\overrightarrow{B}$$

and the vacuum Maxwell equations as

$$\begin{aligned} curl\,\overrightarrow{W} &= i\partial_t\,\overrightarrow{W}, \\ div\,\overrightarrow{W} &= 0. \end{aligned}$$

If we allow the four Minkowski coordinates $x^a = (x, y, z, t)$ to take on complex values, i.e., as $z^a = x^a + iy^a$, we can consider the Maxwell equations as complex equations 'living' on complex Minkowski space. A solution $\overrightarrow{W}(z^a)$ can be understood as a real Maxwell field on the real space by first using $y^a = 0$ and then taking the real and imaginary parts of $\overrightarrow{W}$ as $\overrightarrow{E}$ and $\overrightarrow{B}$.

Though any real analytic Maxwell field can be reinterpreted by analytic extension to $\overrightarrow{W}(z^a)$, we consider a special class of Maxwell fields that *first* have an intrinsic meaning in the complex sense and then see what meaning they have when translated to the real fields. Specifically, we consider the generalization of Lienard-Wiechert fields, {i.e., retarded fields whose source is an electric monopole moving on an arbitrary time-like worldline,} to those with the source moving on an arbitrary *complex analytic* world-line $(z^a = \xi^a(\varphi))$ in *complex* Minkowski space. These fields (including their real restriction) will be referred to as Complex Lienard-Wiechert fields.

When viewed as real Maxwell fields they have a series of interesting features;

1. In addition to an electric monopole moment or charge, q, they possess a time-varying magnetic dipole moment that is given *essentially* by how far the complex world-line is from the real space. The fields are asymptotically 'flat' in the sense of being radiation fields (with both electric and magnetic dipole radiation), falling off as r^{-1}. Though (with a non-vanishing q) an electric dipole moment can be transformed away by an origin shift, the magnetic dipole can not be removed; i.e., it is intrinsic.

2. The following remarkably pretty geometric structure is associated with and defines these fields. One of the real eigenvectors (a principle null vector), l_a, of the Maxwell tensor, *i.e.*, $F^{ab}l_b = \lambda l^a$, turns out to be (a) the tangent vector to a (regular) null geodesic congruence and (b) the congruence is a shear-free congruence. When the twist of the congruence vanishes, the solutions reduce to the real Lienard-Wiechert fields but when the twist is non-vanishing they are the complex Lienard-Wiechert fields.

It appears as if these properties of the Maxwell field remain true and even get extended to the gravitational field when one examines the algebraically-special twisting Einstein-Maxwell equations.

[This work was done in collaboration with Simonetta Frittelli, Duquesne University, Pgh., PA]