PHY 212 General Physics II - Electricity, Magnetism and Light
Summer 2007

Quiz 7 Thursday, Aug 02

Name: WORKED OUT COPY

1. (6 points) Current in an L-R series circuit with an emf $\mathcal{E}$ and time constant $\tau = L/R$ is

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \quad (1)$$

(i) Current in the circuit at time $t = 0$ is:

$i(t = 0) = \frac{\mathcal{E}}{R}(1 - e^{0}) = \frac{\mathcal{E}}{R}(1 - 1) = 0$

(ii) Current in the circuit after one time constant τ is:

$i(t = \tau) = \frac{\mathcal{E}}{R}(1 - e^{-\tau/\tau}) = \frac{\mathcal{E}}{R}(1 - \frac{1}{e})$

(iii) Current in the circuit as time $t \to \infty$ is:

$i(t \to \infty) = \frac{\mathcal{E}}{R}(1 - e^{-\infty}) = \frac{\mathcal{E}}{R}(1 - 0) = \frac{\mathcal{E}}{R}$

2. (9 points) The instantaneous charge q in the capacitor of an L-C circuit is given by

$$q = Q\cos(\omega t + \phi) \quad (2)$$

where $\omega = \sqrt{1/LC}$. Show that (i) $\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C}$ and (ii) $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$.

(i) $i = \frac{dq}{dt} = -\omega Q \sin(\omega t + \phi)$

$$\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{1}{2}L\omega^2 Q^2 \sin^2(\omega t + \phi) + \frac{Q^2\cos^2(\omega t + \phi)}{2C}$$

$$= \frac{1}{2}L\frac{1}{LC}Q^2\sin^2(\omega t + \phi) + \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

$$= \frac{Q^2}{2C}\left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right]$$

$$= \frac{Q^2}{2C}$$

(ii) $\frac{d^2q}{dt^2} = -\omega^2 Q\cos(\omega t + \phi) = -\frac{1}{LC}q$

$\therefore \frac{d^2q}{dt^2} + \frac{1}{LC}q = -\frac{1}{LC}q + \frac{1}{LC}q = 0$