HOMEWORK 7 SOLUTIONS

1. (27.70) (a) During acceleration of ions:

$$qV = \frac{1}{2}mv^2$$

$$v = \sqrt{2qV/m}$$

In the magnetic field: $R = \frac{mv}{qB} = m\frac{\sqrt{2qV/m}}{qB}$

$$m = q\frac{B^2R^2}{2V}$$

(b) $V = q\frac{B^2R^2}{2m} = 2.26 \times 10^4\ V$

(c) The ions are separated by the differences in their diameters.

$$D = 2R = 2\sqrt{\frac{2Vm}{qB^2}}$$

$$\Delta D = D_{14} - D_{12} = 2\sqrt{\frac{2Vm}{qB^2}}\Bigg|_{14} - 2\sqrt{\frac{2Vm}{qB^2}}\Bigg|_{12}$$

$$= 2\sqrt{\frac{2V(1\ amu)}{qB^2}}\left(\sqrt{14} - \sqrt{12}\right)$$

$$= 8.01 \times 10^{-2}\ m \simeq 8\ cm$$ – easily distinguishable.

2. (28.25) There's no contribution from the straight wires. We have two oppositely oriented contributions from the two semicircles:

$$B = (B_1 - B_2) = \frac{1}{2}\left(\frac{\mu_0}{2R}\right)|I_1 - I_2|$$ into the page.

Note that if the two currents are equal, the magnetic field goes to zero at the center of the loop.

3. (28.55) a) If the magnetic field at point P is zero, then from fig. (28.46), the current I_2 must be out of the page, in order to cancel the field from I_1. Also:

$$B_1 = B_2 \Rightarrow \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0 I_2}{2\pi r_2} \Rightarrow I_2 = I_1 \frac{r_2}{r_1} = 2.0\,A$$

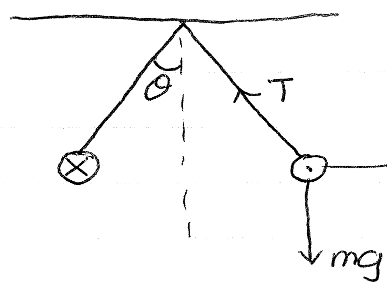
b) Given the currents, the field at Q points to the right and has magnitude

$$B_Q = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right) = \frac{\mu_0}{2\pi}\left(\frac{6A}{0.5m} - \frac{2A}{1.5m}\right)$$
$$= 2.13 \times 10^{-6}\,T$$

c) The magnitude of the field at S is given by the sum of the squares of the two fields because they are at right angles. So:

$$B_S = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi}\left(\left(\frac{I_1}{r_1}\right)^2 + \left(\frac{I_2}{r_2}\right)^2\right) = 2.1 \times 10^{-6}\,T$$

4. (28.61)



The wires are in equilibrium, so:

x: $F = T\sin\theta$ and y: $T\cos\theta = mg$

$$\Rightarrow F = I l B = T\sin\theta = mg\tan\theta \Rightarrow I = \frac{mg\tan\theta}{lB}$$

$$\text{But } B = \frac{\mu_0 I}{2\pi r} \Rightarrow I = \frac{2\pi r\, mg\tan\theta}{l \mu_0 I} \Rightarrow I = \sqrt{\frac{2\pi r\, mg\tan\theta}{l\mu_0}}$$

And $r = [2(0.04\,m)\sin(600°)] = 8.36\times10^{-3}\,m$

$$\Rightarrow I = \sqrt{\frac{2\pi(8.36\times10^{-3}\,m)(0.0125\,kg/m)(9.8\,m/s^2)\tan 6°}{\mu_0}}$$

$$= 23.2\,A$$

5. (28.66) A wire of Length l produces a field,

$$B = \frac{\mu_0 I}{4\pi}\frac{l}{x\sqrt{x^2+(\frac{l}{2})^2}}.$$

Here all edges produce a field into the page so we can just add them up:

$$x = \frac{a}{2} \text{ and } l = b \Rightarrow B_{Left} = \frac{\mu_0 I}{4\pi}\frac{b}{(\frac{a}{2})\sqrt{(\frac{a}{2})^2+(\frac{b}{2})^2}}$$

$$= \frac{\mu_0 I}{\pi}\left(\frac{b}{a}\right)\frac{1}{\sqrt{a^2+b^2}}$$

$$x = \frac{b}{2} \text{ and } l = a \Rightarrow B_{top} = \frac{\mu_0 I}{4\pi}\frac{a}{\sqrt{(\frac{b}{2})^2+(\frac{a}{2})^2}}$$

$$= \frac{\mu_0 I}{\pi}\left(\frac{a}{b}\right)\frac{1}{\sqrt{a^2+b^2}}$$

And the right and bottom edges just produce the same contribution as the left and top, respectively.

Thus the total magnetic field is:

$$B = \frac{2\mu_0 I}{\pi}\left(\frac{b}{a}+\frac{a}{b}\right)\frac{1}{\sqrt{a^2+b^2}} = \frac{2\mu_0 I}{\pi ab}\sqrt{a^2+b^2}.$$

6. (28.69) a) $I = \int_S J\,dA = \int_S (\alpha r)(r\,dr\,d\theta)$

$$= \alpha\, 2\pi\int_0^R r^2\,dr = \frac{2\pi\alpha R^3}{3} \Rightarrow \alpha = \frac{3I}{2\pi R^3}.$$

b) (i) $r \leq R \Rightarrow I_{encl} = \frac{3I}{2\pi R^3} \int_S r^2 dr d\theta$

$$= \frac{3I}{2\pi R^3} \, 2\pi \int_0^r r^2 dr = \frac{I r^3}{R^3}$$

$$\Rightarrow \oint \bar{B} \cdot d\bar{l} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 \left(I \frac{r^3}{R^3} \right)$$

$$\Rightarrow B = \frac{\mu_0 I r^2}{2\pi R^3}$$

(ii) $r \geq R \Rightarrow I_{encl} = I \Rightarrow \oint \bar{B} \cdot d\bar{l} = B\, 2\pi r = \mu_0 I_{encl}$

$$= \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$