HOMEWORK 4 SOLUTIONS

1. (23.36) a) $V = Ed = 18.2\,V$

b) The higher potential is at the positive sheet

c) $E = \frac{\sigma}{\epsilon_0} \Rightarrow \sigma = \epsilon_0 E = 4.25 \times 10^{-9}\ C/m^2$

2. (23.53) a)
$$U = kq^2\left[-\frac{3}{d} + \frac{3}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right] + kq^2\left[-\frac{2}{d} + \frac{3}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right]$$
$$+ kq^2\left[-\frac{2}{d} + \frac{2}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right] + kq^2\left[-\frac{1}{d} + \frac{2}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right]$$
$$+ kq^2\left[-\frac{2}{d} + \frac{1}{\sqrt{2}d}\right] + kq^2\left[-\frac{1}{d} + \frac{1}{\sqrt{2}d}\right] + kq^2\left[-\frac{1}{d}\right]$$
$$\Rightarrow U = kq^2\left(-\frac{12}{d} + \frac{12}{\sqrt{2}d} - \frac{4}{\sqrt{3}d}\right)$$
$$= -\frac{12kq^2}{d}\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right) = -1.46\,q^2/\pi\epsilon_0 d$$

b) Electric potential energy, $U < 0 \Rightarrow$ it is energetically favourable for the crystal ions to be together.

3. (24.4) $\Delta V = Ed = \frac{\sigma}{\epsilon_0} d = 1.14\,mV$

4. (24.12) a) For two concentric spherical shells, the capacitance is:

$$C = \frac{1}{k}\left(\frac{r_a r_b}{r_b - r_a}\right) \Rightarrow kCr_b - kCr_a = r_a r_b$$
$$\Rightarrow r_b = kCr_a / kC - r_a$$

$\Rightarrow \; r_b = 0.175\,m$

b) $V = 220\,V$ and $Q = CV = 2.55 \times 10^{-8}\,C$

5. (24.24) a) $V = Q/C = 2772\,V$

b) Since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to $5544\,V$.

c) $U = \frac{1}{2}CV^2 = 3.53 \times 10^{-3}\,J$. If the separation is doubled, the capacitance halves, and the energy stored doubles. So, the amount of work done to move the plate equals the difference in energy stored in the capacitor, which is $3.53 \times 10^{-3}\,J$.

6. (24.63) a) Reducing the farthest right Leg yields,

$$C = \left(\frac{1}{6.9\mu F} + \frac{1}{6.9\mu F} + \frac{1}{6.9\mu F}\right)^{-1}$$

$$= 2.3\mu F = C_1/3$$

It combines parallel with a $C_2 \Rightarrow C = 4.6\mu F + 2.3\mu F$
$= 6.9\mu F = C_1$

So the next reduction is the same as the first:

$C = 2.3\mu F = C_1/3$.

And the next is the same as the second, Leaving $3C_1$'s in series so $C_{eq} = 2.3\mu F = C_1/3$.

b) For the three capacitors nearest points a and b
$Q_{C_1} = C_{eq}V = 9.7 \times 10^{-4}\,C$

and $Q_{C_2} = C_2V_2 = 6.44 \times 10^{-4}\,C$

c) $V_{cd} = \frac{1}{3}\left(\frac{420}{3}\,V\right) = 46.7\,V$, since by symmetry the total voltage drop over the equivalent capacitance of the part of the circuit from the junctions between a,c and d,b is $\frac{420}{3}$ V, and the equivalent capacitance is that of three equal capacitors C_1 in series. V_{cd} is the voltage over just one of those capacitors, ie $\frac{1}{3}$ of $\frac{420}{3}$ V.