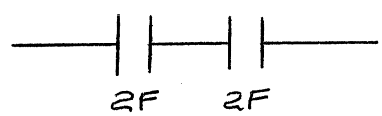
PHY 212 General Physics II - Electricity, Magnetism and Light
Summer 2007

Exam 2 Monday, July 30

Name: WORKED OUT COPY

Some useful constants:
Charge of an electron, $e = 1.6021 \times 10^{-19}$ C
Mass of an electron, $m_e = 9.11 \times 10^{-31}$ kg
Mass of a proton, $m_p = 1.67 \times 10^{-27}$ kg
$\epsilon_0 = 8.854 \times 10^{-12}$ C^2/N m^2
$\mu_0 = 4\,\pi \times 10^{-7}$ T m /A

1. (2 points) Two 2 F capacitors are connected as shown. What is the capacitance of the combination?
   (a) 4 F
   (b) 3 F
   (c) 2 F
   (d) 1 F ✓



$\frac{1}{C_{eq}} = \frac{1}{2F} + \frac{1}{2F} = \frac{1}{1F}$ $\therefore C_{eq} = 1F$

2. (2 points) A positively charged particle with velocity $\vec{v} = +v\,\hat{i}$ enters a magnetic field $\vec{B} = +B\,\hat{k}$. The magnetic force $\vec{F}$ is: (v > 0, B > 0 and F > 0)
   (a) + F $\hat{i}$
   (b) - F $\hat{j}$ ✓
   (c) + F $\hat{k}$
   (d) - F $\hat{i}$

$\vec{F} = q(\vec{v} \times \vec{B}) = qvB(\hat{i} \times \hat{k}) = qvB(-\hat{j}) = -F\hat{j}$

3. (2 points) Motion of a charged particle under the action of a magnetic field alone is always motion with constant
   (a) displacement
   (b) speed ✓
   (c) acceleration
   (d) velocity

4. (2 points) Charged partices pass through a velocity selector without being deflected by the fields when the speed
   (a) $v = E/B$ ✓
   (b) $v = B/E$
   (c) $v = 2E/B$
   (d) $v = B/2E$

5. (12 points) **Human Cells.** Some cell walls in the human body have a layer of negative charge on the inside surface and a layer of positive charge of equal magnitude on the outside surface. Suppose that the surface charge densities are $\pm\ 0.50 \times 10^{-3}$ C/m^2, the cell wall is 5.0×10^{-9} m thick, and the cell wall material has a dielectric constant $K = 5.4$. (i) Find the magnitude of $\vec{\mathbf{E}}$ in the wall between the two charge layers. (ii) Find the potential difference between the inside and outside of the cell. Which is at higher potential? (iii) A typical cell in the human body has volume 10^{-16} m^3. Estimate the total electric field energy stored in the wall of a cell of this size. (*Hint* : Assume that the cell is spherical, and calculate the volume of the cell wall.)

(i) $E = \dfrac{\sigma}{K\epsilon_0} = \dfrac{0.5 \times 10^{-3}\ \text{C/m}^2}{(5.4)\,\epsilon_0} = 1 \times 10^{7}\ \text{V/m}$

(ii) $V = Ed = (1.0 \times 10^{7}\ \text{V/m})(5.0 \times 10^{-9}\ \text{m}) = 0.052\ \text{V}.$

The outside is at the higher potential.

(iii) volume $= 10^{-16}\ \text{m}^3 \Rightarrow R \simeq 2.88 \times 10^{-6}\ \text{m}$

$\Rightarrow$ shell volume $= 4\pi R^2 d = 4\pi (2.88 \times 10^{-6}\ \text{m})^2 (5.0 \times 10^{-9}\ \text{m})$

$= 5.2 \times 10^{-19}\ \text{m}^3$

$\Rightarrow U = uV = (\tfrac{1}{2} K \epsilon_0 E^2)\, V = (\tfrac{1}{2})(5.4)\,\epsilon_0\,(1.0 \times 10^{7}\ \text{V/m})^2 \times (5.2 \times 10^{-19}\ \text{m}^3)$

$= 1.36 \times 10^{-15}\ \text{J}$

6. (5 points) A metallic wire has a diameter of 4.12 mm. When the current in the wire is 8.00 A, the drift velocity is 5.40×10^{-5} m/s. What is the density of free electrons in the metal?

The cross-sectional area of the wire is

$$A = \pi r^2 = \pi (2.06 \times 10^{-3} \text{m})^2 = 1.33 \times 10^{-5} \text{m}^2$$

The current density is

$$J = I/A = 8.00\text{A}/1.333 \times 10^{-5} \text{m}^2 = 6.00 \times 10^5 \text{A/m}^2$$

We have $v_d = J/ne$

$$\therefore n = \frac{J}{v_d e} = \frac{6.00 \times 10^5 \text{ A/m}^2}{(5.4 \times 10^{-5} \text{ m/s})(1.60 \times 10^{-19} \text{ c/electron})}$$

$$= 6.94 \times 10^{28} \text{ electrons/m}^3$$

7. (6 points) When switch S in Fig.1 is open, the voltmeter V of the battery reads 3.08 V. When the switch is closed, the voltmeter reading drops to 2.97 V, and the ammeter A reads 1.65 A. Find the emf, the internal resistance of the battery, and the circuit resistance R. Assume that the two meters are ideal, so they don't affect the circuit.

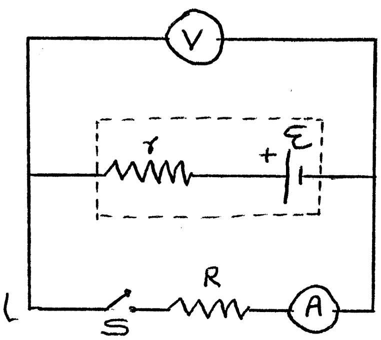


Fig. 1.

When there is no current flowing, the voltmeter reading is simply the emf of the battery : $\mathcal{E} = 3.08\,V$

The voltage over the internal resistance is

$$V_r = 3.08\,V - 2.97\,V = 0.11\,V \Rightarrow r = \frac{V}{I} = \frac{0.11\,V}{1.65\,A} = 0.067\,\Omega$$

$$V_R = 2.97\,V = (1.65\,A)\,R$$

$$\therefore R = \frac{2.97}{1.65\,A} = 1.8\,\Omega$$

8. (6 points) A battery-powered global positioning system (GPS) receiver operating on 9.0 V draws a current of 0.13 A. How much electric energy does it consume during $1\frac{1}{2}$ h?

$$W = Pt = IVt = (0.13\,A)(9V)(1.5)(3600s)$$

$$= 6318\ J.$$

9. (10 points) Four resistors and a battery of negligible internal resistance are assembled to make the circuit in Fig.2. Find (i) the equivalent resistance of the network; (ii) the current in each resistor.

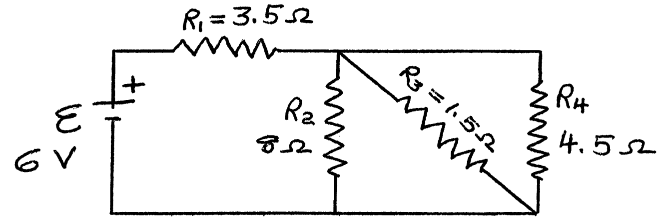


Fig. 2

(i) The three resistors R_2, R_3 and R_4 are in parallel, so:

$$R_{234} = \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1}$$

$$= \left(\frac{1}{8.2\,\Omega} + \frac{1}{1.5\,\Omega} + \frac{1}{4.5\,\Omega}\right)^{-1} = 0.99\,\Omega$$

$$\Rightarrow R_{eq} = R_1 + R_{234} = 3.5\,\Omega + 0.99\,\Omega = 4.49\,\Omega$$

(ii) $I_1 = \frac{\mathcal{E}}{R_{eq}} = \frac{6.0\,V}{4.49\,\Omega} = 1.34\,A \Rightarrow V_1 = I_1 R_1 = (1.34\,A)(3.5\,\Omega)$

$= 4.69\,V$

$$\Rightarrow V_{R_{234}} = I_1 R_{234} = (1.34\,A)(0.99\,\Omega) = 1.33\,V$$

$$\Rightarrow I_2 = \frac{V_{R_{234}}}{R_2} = \frac{1.33\,V}{8.2\,\Omega} = 0.162\,A$$

$$I_3 = \frac{V_{R_{234}}}{R_3} = \frac{1.33\,V}{1.50\,\Omega} = 0.887\,A$$

$$I_4 = \frac{V_{R_{234}}}{R_4} = \frac{1.33\,V}{4.5\,\Omega} = 0.296\,A.$$

10. (8 points) In an experiment with cosmics rays, a vertical beam of particles that have charge of magnitude $3e$ and mass 12 times the proton mass (m_p) enters a uniform horizontal magnetic field of 0.250 T and is bent in a semicircle of diameter 95.0 cm as shown in Fig. 3. (i) Find the speed of the particles and the sign of their charge. (ii) How does the speed of the particels as they enter the field compare to their speed as they exit the field?

(i) $R = \frac{mv}{qB}$

$v = \frac{qBR}{m} = \frac{3(1.6\times10^{-19})(0.25)\times\left(\frac{0.95}{2}\right)}{12(1.67\times10^{-27})}$

$= 2.84\times10^{6}$ m/s

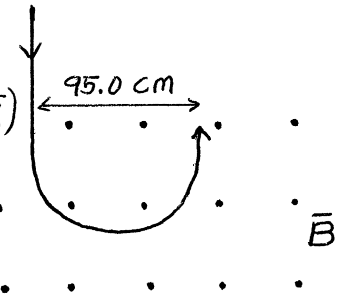


Fig. 3.

Since $\bar{v}\times\bar{B}$ is to the ~~right~~ Left but the charges are bent to the right, they must be negative.

(ii) $F_{grav} = mg = 12(1.67\times10^{-27})(9.8) = 1.96\times10^{-25}$ N

$F_{magnetic} = qvB = 3(1.6\times10^{-19})(2.84\times10^{6})(0.25)$

$= 3.41\times10^{-13}$ N

Since, $F_{magnetic} \simeq 10^{12} F_{grav}$, we can safely neglect gravity.

(iii) The speed does not change since the ~~ab~~ magnetic ~~field is~~ force is perpendicular to the volocity and ~~does~~ therefore does not do work on the particles.

11. (5 points) In Fig.4, a wire carrying current into the plane of the figure is between the north and south poles of two bar magnets. What is the direction of the force exerted by the magnets on the wire?

$\bar{F} = I \bar{l} \times \bar{B}$

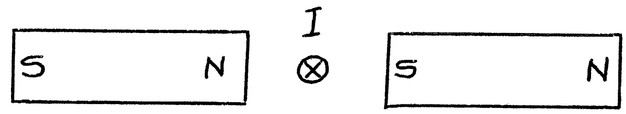


Fig. 4

Between the poles of the magnet, the magnetic field points to the right. Using the finger tips of your right hand, rotate the current vector by 90° into the direction of the magnetic field vector. Your thumb points downward - which is the direction of magnetic force.