

**PHY662 - Quantum Mechanics I**      **Name** \_\_\_\_\_  
**Final Exam, 5:30 PM, Monday, May 3, 2004**

NO CALCULATORS. CLOSED BOOK. Write out your work and answers on the paper provided. Please properly justify your answer. The total number of points available on this exam is 50. You have two hours to complete this exam.

**There are two pages on this cover sheet of formulas.**

$R(\vec{\theta}) = e^{-i\vec{\theta}\cdot\vec{S}/\hbar}$	$\gamma = \frac{gq}{2mc}; g_{\text{electron}} \approx 2$
$[S_i, S_j] = i\hbar\epsilon_{ijk}S_k$	$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$ [for spin- $\frac{1}{2}$ ]
$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$J_- jm\rangle = \sqrt{j(j+1) - m(m-1)} j, m-1\rangle$ $J_+ jm\rangle = \sqrt{j(j+1) - m(m+1)} j, m+1\rangle$
$H_{\text{dipole}} = -\vec{\mu} \cdot \vec{B}; \vec{\mu} = \gamma\vec{S}$ $[\sigma_i, \sigma_j]_+ = 2\delta_{ij}$	$J^2 j, m\rangle = j(j+1)\hbar^2 j, m\rangle$ $J_z j, m\rangle = m\hbar j, m\rangle$
$\frac{m_{\text{proton}}}{m_{\text{electron}}} = 1836.15\dots$	$(\hat{n} \cdot \vec{\sigma})^2 = 1$ [ $\hat{n}$ is a unit vector]
$g_{\text{proton}} \approx 5.6; \gamma_{\text{neutron}} = \frac{(-3.8)e}{2M_{\text{neutron}}c}$	$\gamma_{\text{proton}} \approx (2\pi)(43 \text{ MHz/T})$
<b>Non-degenerate, time-independent perturbation theory:</b> $E_n^1 = \langle \psi_n^0   H'   \psi_n^0 \rangle$ $ \psi_n^1\rangle = \sum_{m \neq n} \frac{\langle \psi_m^0   H'   \psi_n^0 \rangle}{E_n^0 - E_m^0}  \psi_m^0\rangle$ $E_n^2 = \sum_{m \neq n} \frac{ \langle \psi_m^0   H'   \psi_n^0 \rangle ^2}{E_n^0 - E_m^0}$	<b>WKB solutions away from the turning points:</b> $\psi(x) = [k(x)]^{-1/2} e^{\pm i \int^x dx' k(x')}$ , $\psi(x) = [\kappa(x)]^{-1/2} e^{\pm \int^x dx' \kappa(x')}$ , $k(x) = \sqrt{\frac{2m}{\hbar^2} [E - V(x)]}$ (classically allowed) $\kappa(x) = \sqrt{\frac{2m}{\hbar^2} [V(x) - E]}$ (classically forbidden)
<b>The Airy function <math>Ai(z)</math> solves the equation <math>\frac{d^2 Ai(z)}{dz^2} = z Ai(z)</math> and has asymptotics listed on the right.</b>	<b>Asymptotics of the Airy function <math>Ai(z)</math>:</b> $Ai(z) \sim (2\sqrt{\pi}z^{1/4})^{-1} e^{-\frac{2}{3}z^{3/2}}, z \gg 0,$ $Ai(z) \sim [\sqrt{\pi}(-z)^{1/4}]^{-1} \sin[\frac{2}{3}(-z)^{3/2} + \frac{\pi}{4}], z \ll 0$

For a spin-1/2 particle in an applied magnetic field  $\vec{B} = B_0\hat{z} + B_1 \cos(\omega t)\hat{x}$ , with the spin precessing at frequency  $\omega_0$  due to the  $z$ -component of  $\vec{B}$ ,

$$|\psi(t)\rangle = e^{i\omega t\sigma_z/2} e^{i\vec{\Omega}t/2} |\psi(0)\rangle, \text{ with } \vec{\Omega} = (\omega - \omega_0)\sigma_z + \omega_1\sigma_x, \text{ where } \omega_1 = \frac{\gamma B_1}{2}.$$

<p>Taking <math>\psi(t) = \sum_n d_n(t)e^{-iE_n t/\hbar} n\rangle</math>,  defining <math>\omega_{fn} = (E_f - E_n)/\hbar</math>,  <math>i\hbar\dot{d}_f(t) = \sum_n d_n(t)\langle f H' n\rangle e^{i\omega_{fn}t}</math>.</p>	<p>For <math>H = H^0 + H'e^{-i\omega t}</math>, Fermi's golden rule is  <math>\Gamma = \frac{2\pi}{\hbar}  \langle f H' i\rangle ^2 \delta(E_f - E_i - \hbar\omega)</math>.</p>
$\begin{aligned}\vec{B} &= \vec{\nabla} \times \vec{A} \\ \vec{E} &= -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \phi \\ \vec{\nabla} \cdot \vec{A} &= 0 \\ \nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} &= 0 \\ u &= \frac{1}{8\pi} (E^2 + B^2)\end{aligned}$	<p>Gauge transformation:</p> $\begin{aligned}\vec{A}(\vec{r}, t) &\rightarrow \vec{A}(\vec{r}, t) - \vec{\nabla} \Lambda(\vec{r}, t) \\ \phi(\vec{r}, t) &\rightarrow \phi(\vec{r}, t) + \frac{1}{c} \frac{\partial \Lambda(\vec{r}, t)}{\partial t} \\ \psi(\vec{r}, t) &\rightarrow e^{-iq\Lambda(\vec{r}, t)/\hbar c} \psi(\vec{r}, t)\end{aligned}$
$H = \frac{1}{2m} \left( \vec{p} - \frac{q}{c} \vec{A} \right)^2 + q\phi.$	$\vec{j}(\vec{r}) = \frac{\hbar}{2i\mu c} \left( \psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^* \right) - \frac{q}{\mu c} \psi^* \psi \vec{A}$
<p>For a harmonic oscillator with Hamiltonian <math>\hbar\omega(a^\dagger a + \frac{1}{2})</math> with eigenstates <math> n\rangle</math>, <math>n = 0, 1, \dots</math>, <math>\langle n a^\dagger a n\rangle = n</math>,</p> $\begin{aligned}a n\rangle &= \sqrt{n} n-1\rangle \\ a^\dagger n\rangle &= \sqrt{n+1} n+1\rangle.\end{aligned}$	$\begin{aligned}[L^2, [L^2, \vec{R}]] &= 2\hbar^2 (\vec{R}L^2 + L^2\vec{R}) \\ [L_z, R_z] &= 0 \\ [L_z, R_x] &= i\hbar R_y \\ [L_z, R_y] &= -i\hbar R_x\end{aligned}$
$\begin{aligned}\frac{e^2}{\hbar c} &= \frac{1}{137.04} \\ a_0 &= 5.29 \times 10^{-11} \text{ m} \\ R_y &= 13.606 \text{ eV} \\ \text{eV} &= 1.602 \times 10^{-19} \text{ J} \\ m_{\text{electron}} &= 9.11 \times 10^{-31} \text{ kg} \\ c &= 3.00 \times 10^8 \text{ m/s} \\ \text{Solar constant} &\approx 1000 \text{ W/m}^2 \\ hc/e &= 4.14 \times 10^{-15} \text{ T/m}^2\end{aligned}$	<p>The Fourier transform of <math>\frac{1}{r}</math> is <math>\frac{4\pi}{k^2}</math>.</p> <p>Ehrenfest's theorem:</p> $\frac{d\langle \Omega \rangle}{dt} = (i\hbar)^{-1} \langle [\Omega, H] \rangle + \left\langle \frac{\partial \Omega}{\partial t} \right\rangle$