PHY662, Spring 2004
Outline for Thurs. Feb. 5, 2004
NMR, MRI, C-G

5th February 2004

1 Miscellaneous

1. Colloquium on Tuesday: comments, questions? Two state systems, definitely. [Further info, see http://arxiv.org/abs/cond-mat/0305461 .]

2. Homework #3 handed back. Generally good work, but some missteps. The high score was 7/10. Try to:

   (a) Back up your assumptions! It may be that $\omega \approx \omega_0$, but that difference could be important. If you want to neglect a term, **do the actual comparison** ($\omega - \omega_0$ vs. ω_1).

   (b) Watch units. This also came up in HWK #2: see the key there. Common combinations, like $\gamma_{\rm proton}$, recur - use a value (43 MHz/T) rather than recomputing each time. The main problem here is **SI vs. CGS units**. The electrodynamics here is in CGS (note the absence of ϵ_0, for example).

   (c) When solving a problem with a constant Hamiltonian, **diagonalize the Hamiltonian** and work with the eigenstates (in the transformed basis). In problem 2, express the "position" eigenstate $|A\rangle$ in terms of the energy eigenstates.

   (d) **Take great care with non-commuting operators.** In particular, remember that $e^{A+B} \neq e^A e^B$. NOTE: please take great care with the expression that I wrote down in class the other day: $U(t) = e^{\int_0^t H(t')dt'/i\hbar}$. You can't simply integrate the Hamiltonian and apply it. This is a formal representation of a product of opertors: $U(t) \approx \prod(1 + \delta t \frac{H(t)}{i\hbar})$, and the operators in this product rarely commute.

2 Review information, Exam #1, Feb. 12

Review your class handouts, your own notes, and understand how to solve each homework problem. Pay attention to the text and reading (Feynman, Baym) where it overlaps with lecture and homework. The exam will be during the class period on Thursday and is to be completed in that amount of time. There will be three (maybe four) multi-part questions, where you will need to do a calculation (without a calculator, just express what divisions you would need to carry out, e.g., an answer might be something like "$(2\pi)(43\,\text{MHz})/(3{\cdot}10^{-3})$", some type of derivation, and an abstract calculation. There will also be short qualitative questions about applications of spin.

2.1 Reminder of topics for the exam

The main focus is the physics of spin-1/2 and its applications. In this context, we studied

- Symmetries in general: conservation laws and generators of symmetry.
- The rotation group.
- Representations of the rotation group with generators $\vec{S}$.
- General manipulation of the rotation generators: raising and lowering operators, Clebsch-Gordon coefficients for representing product representations.
- The Pauli matrices and their properties and their use in representing the spin-1/2 generators.
- How to obtain an arbitrary rotation by composing sequences of rotations about x- and z- axes.
- Quantum cryptography: one-time pads and quantum key distribution.
- Precession of a spin-1/2 particle in an external magnetic field.
- Magnetic resonance: adding a periodic term to the Hamiltonian and solving the equations of motion.
- Solving two state systems with time-varying (periodic) potentials using magnetic resonance techniques.
- Magnetic resonance imaging: tricks for finding spatial information using magnetic resonance.

2.2 Draft list of formulas and constants that will be available on the Exam.

This is a set of formulas and constants that are **intended to help you avoid sign errors or simple slips** that would affect your answers. You are expected to know the Schrodinger equation.

$$R(\vec{\theta}) = e^{-i\vec{\theta}\cdot\vec{S}/\hbar}$$

$$[S_i, S_j] = i\hbar\epsilon_{ijk}S_k$$

[Insert formulas for rotation in polar coordinates]

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} ; \sigma_x = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} ; \sigma_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$[\sigma_i, \sigma_j]_+ = 2\delta_{ij}$$

$$\mathcal{H}_{\text{dipole}} = -\vec{\mu}\cdot\vec{B} \, ; \, \vec{\mu} = \gamma\vec{S}$$

$$\gamma = \frac{gq}{2mc}; \, g_{\text{electron}} \approx 2 \, ; \, g_{\text{proton}} \approx 5.1 \, ; \, \gamma_{\text{neutron}} = \frac{(-3.8)e}{2M_{\text{neutorn}}c}$$

$$m_{\text{proton}}/m_{\text{electron}} \approx 1800$$

$$\gamma_{\text{proton}} \approx (2\pi)(43\,\text{MHz/T})$$

$$|\psi(t)\rangle = e^{i\omega t\sigma_z/2}e^{i\vec{\Omega}t/2}|\psi(0)\rangle \, , \, \vec{\Omega} = (\omega - \omega_0)\sigma_z + \omega_1\sigma_z \, ; \omega_1 = \frac{\gamma B_1}{2}$$