PHY662, Spring 2004
Outline for Tues. Jan. 13, 2004

13th January 2004

Introduction

- Course syllabus - review in detail. Pay special attention to homework procedures and practice with collaboration and research.
- Summarize calendar.
- Reading assignment for Thursday: Feynman, Leighton, and Sands, Ch. 5 (skim latter part of Ch. 5), Secs. 6-1 through 6-3. Shankar, pp. 373-385. Note that they use very different approaches. (Feynman is also *building* the ideas of representations and transformations, while Shankar already has that technology available for discussion.)
- It has been a long time since I have had to use a lot of quantum mechanics. So I will be grateful for corrections and input.
- Physics is based on observation - *must not forget this.*
- Mathematics is a surprisingly successful method for organizing our observations. The tools developed to study QM are of great use in many different areas. Concepts guide mathematics and mathematics suggests concepts.
- *Thought experiments* can be of high importance: explore possible results, draw conclusions about the logical structure of a theory.
- Quantum mechanics - introductory remarks.
  - Is, like all physical theories, incorrect.
  - But it is amazing that a linear theory (in the sense of superposition of states) works so well and is the basis for so much else (quantum field theory, etc.)
  - Spin is a realm which has no (single-particle) classical limit, though there are parallels.

- **One can deduce *a lot* from symmetries, continuity, and linearity**. *We will be using this for spin, especially.*
- Spin-1/2 is most important case (photon close second). Very general, also, e.g., two-level systems.
- One measures probabilities, not amplitudes. But (relative) amplitudes can be deduced from probabilities. That is amazing, but also good - otherwise why would we need amplitudes to describe experiments?

Starting Spin

Conceptual background for describing spin, experiments indicating its existenc, and start of building a consistent mathematical description.

1. Rotations: transformations on our space and the objects in it.

   (a) One type of spatial transformation: translations. Do they commute?

   (b) What is P? (reminder) Does P commute with translations?

   (c) What is $SO(3)$? Do rotations commute?

   (d) What is the *global* structure of $SO(3)$? What is its dimensionality? [What is the dimensionality of $SO(4)$?] Rotation by 2π about a given axis does what to a vector?

   (e) Experiment in physical space on a sequence of rotations. Map a sequence of rotations $R(t)$ to a twisted object. Gives information on *connectedness* of the set of rotations.

   (f) Also use example of 4π rotation as mentioned in Wald's text on general relativity:

      i. Rotate by 4π about the z-axis: this is a sequence $R(t)$, with $R(t) = R(4\pi t\hat{z})$. $R(0) = R(1/2) = R(1) = I$.

      ii. Hold first half of sequence fixed, modify second half while maintaining $R(1/2) = R(1) = I$, by slowly changing the rotation axis from $\hat{z}$ to $-\hat{z}$.

      iii. Cancel out first half with second half, to get identity sequence.

   (g) Continuity gives constraints on the physical effect of a sequence of rotations that is related to the trivial sequence. What about the phase, for example? *Spinors* will provide a representation of $SO(3)$ where a 2π rotation will not be equal to the identity in phase.

2. Representations of the rotation group.

   (a) Mathematical description of the effect of rotations.

   (b) Orbital angular momentum $\vec{L} = \vec{R} \times \vec{P}$. This is derived from the operations on $\psi(x, y, z)$ that generate rotations.

(c) Other representations: divided according to angular momentum quantum number j. $\vec{J}$ are the *generators* of rotations. This is an idea that extends beyond QM.

(d) Direct products, e.g., of "internal coordinates" and spatial coordinates. Wave function is a function of x, y, z, s_z, color charge, etc. These give the amplitudes to find a particle at a particular position, with a given spin, "color", etc. Notation: $\psi(x, y, z, s_z; t)$, $|x, y, s_z\rangle$.

3. Measurement of spin or the magnetic moment.

   (a) The S-G apparatus allows us to separate particles according to magnetic moment. Classically, magnetic moments come from currents and so magnetic moments are associated with current loops. Here, there is no spin current. (Classical "spin" is really a sum of the angular momenta of the components of an extended object.)

   (b) Stern-Gerlach apparatus a la Feynman, et al. [Note that Feynman starts with S-G, while Shankar ends with S-G.]

   (c) Assume two dimensional space of states. This can be derived assuming special relativity and a linear equation differential equation for the wave function: a 4-component wave function arises naturally (Dirac spinor) which, in the non-relativistic limit, can be approximated by a 2-component classical spinor with all of the properties we will assume here.

   (d) Algebra of spin-1/2 operators is from $SO(3)$. Shankar uses the machinery of this algebra to build up the calculation of amplitudes (for example, for a spin to point in a particular direction). Feynman uses continuity and rotation symmetry in a more hidden fashion, directly related to S-G apparatus, to get the same results, deriving projection operators and identities along the way. So it is a useful review.

   (e) Work through some of Feynman's conclusions, today. Later will go to Shankar description.

   (f) Measure probabilities - must be same if the spin is oriented in same direction. *But what about amplitudes? They can depend on rotation / frame of reference.*

      i. Rotations about the z-axis (FLS 6-3): maintain magnitude of probabilities as "up" is still "up" in a rotated frame. By convention, share the relative phase change between "up" and "down" between the two states.

      ii. How do the phases depend on angle? Use composition of rotations to get linearity.

      iii. What about a 2π rotation? *Check what happens to the x-eigenstates!* [Think about other cases, e.g., spin-1.]