PHY662 - Quantum Mechanics II
HWK #7, Due Tues., Mar. 2, at the *start* of class

- Reading: Read Ch. 8 of Griffiths, *Introduction to Quantum Mechanics.*

1. *Checking last week's answer to problem #2.* Consider the potential

$$V(x) = \begin{cases} \infty & x < -a \\ F|x| & -a < x < a \\ \infty & a < x \end{cases} ,$$

which you studied last week using the WKB approximation. Let the constant F be small in some sense, so that you can compute a perturbation series expansion for the ground and first excited states for a particle of mass m in this potential.

(a) What are the $F = 0$ energy eigenstates for this potential?

(b) Compute the first order (in F) estimates for the energies of the ground state and the first excited state, using time-independent perturbation theory.

(c) Write down the ground state energy E_0 and the first excited state E_1, to 6 significant figures (within this approximation), when $F = \kappa\frac{\hbar^2\pi^2}{8ma^3}$, for the two cases $\kappa = 0.2$ and $\kappa = 0.5$, to first order in F. That is, compute $E_0(\kappa = 0.2)$, $E_1(\kappa = 0.2)$, $E_0(\kappa = 0.5)$, and $E_1(\kappa = 0.5)$.

(d) Compare your results from perturbation theory with the WKB results from last week's problem.

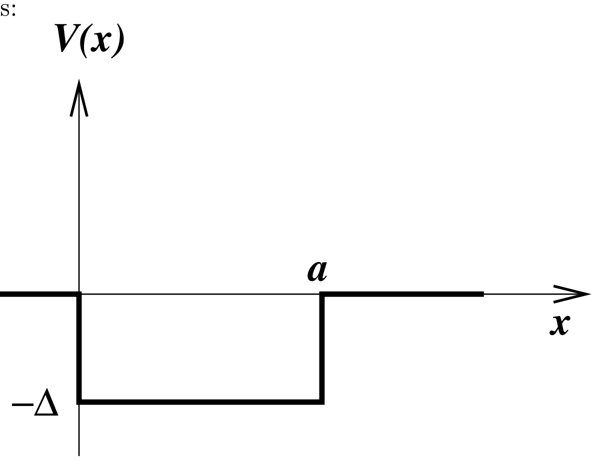
(e) Write out the expression for the sums you would need to compute to find the second order correction to E_0 and E_1. You do not need to evaluate these sums, but simplify the terms in the sum as much as possible.

2. *Extending last week's answer to problem #1: Airy functions and perturbation theory.* Let's consider the problem from last week, where the spectrum of mesons can be estimated using a linear potential for $q\bar{q}$ pairs. The potential was taken to be $V^0(r) = V_0 + Kr$. In this problem, you will compute a new energy estimate, using second-order perturbation theory for a modified potential. Use the data from last week's problem and the key to that problem where necessary.

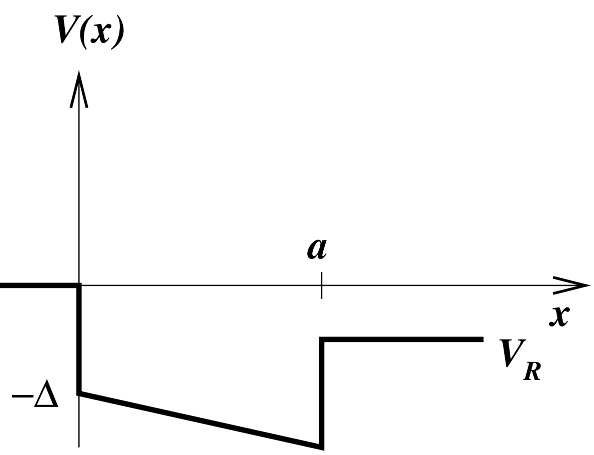
(a) *Effect of a $\delta(r)$ change to the potential.* Fix up the energy estimates by trying a different model. At close distances the quarks attract more strongly. This is often described by a Coulombic attraction. Here, try something simpler: add a δ-function attraction $V^1(x) = -u\delta(r)$ to the potential. That is, the total potential is $V(x) = V^0(x) + V^1(x)$. To first order in u, what are the corrections to the energies of the first four radial ($\ell = 0$, $n = 1, 2, 3, 4$) states?

(b) By varying u, how well can you match the spectrum 9.460, 10.023, 10.355, 10.580 GeV/c^2 for the $n = 1, 2, 3, 4$ S states of $b\bar{b}$ using your results to first order in u?

3. *Tunneling amplitudes for an electron.* Quantum mechanics is central to understanding the behavior of small ("mesoscopic" or even "nanoscopic") electronic devices. Fabrication techniques now allow one to play all sorts of games with potentials. Consider an electron moving in a narrow wire. Assume that the wire is narrow enough that its motion is only along one dimension. If the wire is metallic, the potential is nearly constant along the wire. Now interrupt the wire with a semiconducting segment of length a that has a potential $-\Delta$ relative to the metallic ends. Then, when the potential at both ends of the wire is zero, a plot of the potential would look like this:



If the potential of the right end of the wire (the right "lead") is lowered, the potential drop will be spread across the semiconducting region, like this:



The potential drop between the left metallic end (where $x < 0$, $V = 0$)

and the right metallic end (where $x > a$, $V = V_R < 0$) is spread out over the insulator. The potential in the insulating region is $-\Delta + V_R(x/a)$. Now let the left end wire be fed electrons, taken to have effective mass m (which is not the mass of the electrons in free space, by the way). A reasonable question, which would determine the current, is what is the probability that the electron from the left will end up propagating off to $x = \infty$? That is, what is the transmission coefficient through the insulating region? Answer this question by taking the following steps.

(a) Let the energy of the electrons coming from the left be E_0. What is the wave function of the incoming electrons in the region $x < 0$? There will be an amplitude B for the wave to be reflected. What is the form of the wave function for these reflected electrons?

(b) In the region $0 < x < a$, what is the wave function, *within the WKB approximation*? This wave function has two coefficients in it that will be determined by the boundary conditions: let C and D be the amplitudes of the two wave functions that make up the total solution.

(c) Let G be the amplitude of the transmitted wave (where $x > a$). Match up your solutions (continuity of $\psi(x)$ and $\psi'(x)$ at $x = 0$ and $x = a$) and solve for the relationship between G and A. Use this result to write the formula for the transmission coefficient. (It is not a very pretty formula; we will plot the results in class.)