

PHY662 - Quantum Mechanics II

HWK #5, Due Tues., Feb. 17, at the *start* of class

- Reading: Read through p. 435 of Shankar by Tuesday, Feb. 17.
- This is a short (5 pt.) homework.

1. *Variational method introduction.*

- (a) *Gaussian integral.* Suppose you had a unnormalized wave function $\psi(\alpha, x) = e^{-\alpha x^2}$, with $\alpha > 0$, with the single spatial coordinate x . What is the normalized wave function?
- (b) *Shankar Problem 16.1.1, spelled out in detail:* Use $\psi(\alpha, x) = e^{-\alpha x^2}$ (properly normalized; with $\alpha > 0$) as a trial ground-state wave function for a single particle of mass m moving in one dimension (x -coordinate). The potential for the particle is the harmonic potential $V = \frac{1}{2}m\omega^2 x^2$. Find the value of α , α_{\min} , that minimizes $\langle H \rangle$. Compare the energy estimate from this variational solution $\psi(\alpha_{\min}, x)$ with the exact solution for the ground-state energy for this potential.
- (c) *Modifying the potential.* Now consider adding a δ -function to the potential to represent a repulsive point at the origin. Let the total potential be $V = \frac{1}{2}m\omega^2 x^2 + u\delta(x)$. Compute the resulting *change* in $\langle H \rangle = \langle \psi(\alpha, x) | H | \psi(\alpha, x) \rangle$ (i.e., the change from the calculation in part (b)). Then, using your work from part (b), write the new full equation for α that gives a minimum for $\langle H \rangle$. Just write out the equation (4th-order polynomial in $\sqrt{\alpha}$) that you would need to solve in order to exactly determine this optimal value for α . *You do not need to solve this equation.*
- (d) *Consistency check.* Study this 4th-order equation in $\sqrt{\alpha}$ *qualitatively*: how does the addition of a repulsive potential at the origin change the estimated ground state wave function? That is, how does α_{\min} for $u > 0$ compare, qualitatively, with α_{\min} for the case $u = 0$? How does this change in α change the width of the probability distribution for the particle?