PHY662 - Quantum Mechanics II
HWK #3, Due Tues., Feb. 3, at the *start* of class

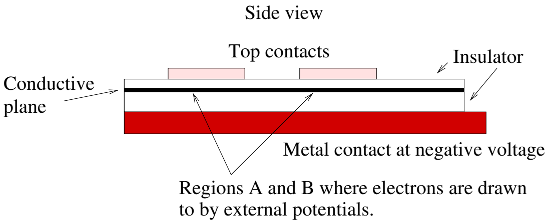
- For Thursday, Feb. 3, read the sections in Shankar at the start of Chapter 15, through about p. 412.
- The reading for Tuesday, Feb. 8, will be given in Thursday's class.
- Reading Feynman's chapters on the ammonia molecule might help some with Problem 2.

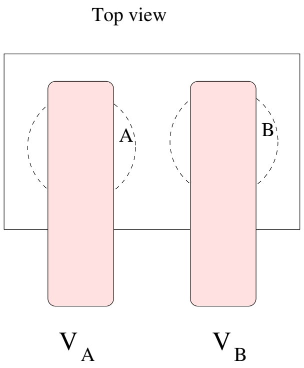
1. *Magnetic resonance with real numbers and an off-resonance drive.* Consider a proton in the $|\downarrow, \hat{z}>$ state. It is immersed in a magnetic field of strength $|\vec{B}| = 1\,\mathrm{T}$ that is oriented in the $\hat{z}$-direction. Let $\omega_0 = \gamma|\vec{B}|$. An oscillating magnetic field of magnitude 20 G at frequency $\nu = \frac{\omega_0}{2\pi} + 0.1\,\mathrm{MHz}$ is then applied for $10\,\mu\mathrm{s}$.

    (a) Why wouldn't it make sense to speak of a pulse at frequency ν that lasted for $10\,\mathrm{ns}$?

    (b) After the $10\,\mu\mathrm{s}$ pulse, what is the probability that the proton's spin is oriented down in the $\hat{z}$-basis? (Ignore the interaction of the proton with other protons and spontaneous radiation - just consider the interaction of the proton with the static and oscillating applied magnetic fields.)

2. *Quantum dots.* Suppose I have an electronic device as drawn below. These types of devices are common in experimental mesoscopic physics experiments. The electrons in the device lie in the conductive layer, which is a very thin material sandwiched between two insulating layers. The material is a semiconductor, so the density of electrons is very low and easily influenced by external potentials. The device is controlled by metal *gates* which control the electrostatic potentials in the conductive layer. There are three gates here: two on top and one on the bottom. The gate on the bottom is at a fixed negative voltage. This bottom gate uses electrostatic repulsion to keep electrons out of the conductive layer, except in the regions A and B: in these regions electrons have low energy due to the positive potentials applied to the two top gates. Low energy electrons are classically forbidden in the other parts of the conductive layer.





Sketch of electronic device – approximate dimensions of order of 1 micrometer.

There are two classical positions for the electron, which can initially be used as basis states, $|A\rangle$ and $|B\rangle$. These positions correspond to local minima of the potential for the electron. [It might help to sketch the potential as a function of position along the horizontal direction in the side view.] The electrostatic potential at these two positions in the conducting layer is nearly equal to the applied voltages V_A and V_B that are controlled by the experimentalist.

(a) What are the diagonal elements $\mathcal{H}_0$ of the Hamiltonian for the electron in this potential, written as a 2×2 matrix using $|A\rangle$ and $|B\rangle$ as a basis?

(b) There is also a term $\mathcal{H}_\infty = \kappa(|A\rangle\langle B| + |B\rangle\langle A|)$ in the Hamiltonian, where κ is a number (possibly complex) with units of energy.

   i. What condition must κ satisfy for $\mathcal{H}_1$ to be Hermitian? [*For the rest of this problem, take $\mathcal{H}_1$ to be Hermitian.*]

   ii. What does the total Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$ look like written out as a 2×2 matrix?

(c) What does $\mathcal{H}_1$ correspond to, physically? Think of how it affects the Schrodinger equation - what type of dynamics does this new term correspond to? Explain in physical terms.

(d) What are the eigenvalues and eigenvectors of $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$, for the case $V_A = V_B$?

(e) Suppose that the initial state of the system is $|A\rangle$ and $V_A = V_B$. What is the state after $\kappa^{-1}\hbar$ units of time?

(f) Suppose that you now add an oscillatory component to V_A. Let this varying voltage be written as $V_A = V_A^0 + (\delta V)\cos(\omega t)$, with $V_A^0 = V_B$. *This perturbation does not only affect $\langle A|\mathcal{H}|A\rangle$, it also can affect κ,* making it time dependent. Physically, why is κ affected by varying V_A?

(g) [Most difficult part and most points on this part.] Have the minimum of the potential at A varying as in (f). Do not use the fact that κ varies with time for this part of this problem. Keep the potential at

B fixed, with $V_B = V_A^0$. All of the time dependence is then just in the variation of the energy of $|A\rangle$. If the particle starts in its ***lowest energy eigenstate of the static potential at time*** $t = 0$, what is the probability of it being in its higher energy eigenstate as a function of time, when $\delta V \neq 0$? You can ignore the 2ω terms that arise in your calculation or any other non-resonant rapidly varying term (e.g., terms with frequency ω that remain even when you translate to the "rotating" frame - this should be a hint to follow the MR derivation, using Pauli matrices, etc., to describe the dynamics of this two-state system). (Another hint: rewrite the perturbation in the basis of energy eigenstates for constant V_A, i.e., $\delta V = 0$.) What happens when $\omega \approx 2\kappa$? Explain quantitatively and qualitatively.

3. *NMR.* Nuclear magnetic resonance is an extremely powerful tool for analyzing materials and chemicals. The radiation emitted by nuclei excited by an RF pulse is analyzed to determine chemical bonds, etc. Explain how this is done using the principal of *chemical shift,* using a brief paragraph or two, written in your own words. What is the chemical shift due to? Indicate your sources in your answer, of course.