PHY662 - Quantum Mechanics II
HWK #2, Due Tues., Jan. 27, at the *start* of class

- Read the section from Baym to be handed out on Thursday.
- Complete Ch. 14 of Shankar, though you can skip pp. 393-395 (sections on "Paramagnetic Resonance" and "Negative Absolute Temperature").

1. *Practice on a spinor.* Make up a state for a spin-$\frac{1}{2}$ particle. This is your very own spinor; it has two amplitudes in it. All I ask is that it is not an eigenstate for S_x or S_y or S_z and that at least one of the amplitudes is complex. So it has a bit of character. Answer the following questions about your spinor:
   (a) Is your spinor properly normalized? If not, show how to normalize it and carry out the rest of the exercise with the normalized spinor.
   (b) What are $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$?
   (c) What are $\langle S_x^2 \rangle$, $\langle S_y^2 \rangle$, and $\langle S_z^2 \rangle$? [Check that the total is $\frac{3}{4}\hbar^2$.]
   (d) What do you get if you apply S_+ to your spinor? What if you apply S_-?
   (e) What direction $\hat{n}$ gives $\langle \hat{n} \cdot \vec{S} \rangle = \hbar/2$? (That is, what direction is your spin pointing in?)
   (f) What would the elements of your spinor be if you rotated it by an angle of $\frac{\pi}{3}$ about the $\hat{z}$-axis?

2. *Using real units.* This problem is an exercise in using real numbers.
   (a) Exercise 14.4.4 in Shankar.
   (b) Repeat (a) using a neutron, rather than an electron.
   (c) Use your spinor from problem #1. Suppose your spinor represents a proton. If you turn on a magnetic field of 0.5 Tesla in the $\hat{x}$-direction how long do you need to leave it on to rotate the spin direction of your proton by $\frac{\pi}{2}$?
   (d) Assume silver atoms are entering a Stern-Gerlach apparatus. They have been heated in an oven to a temperature of 1200 K.
      i. What is the average speed of the silver atoms?
      ii. If these atoms pass through a S-G apparatus of length 1m, with a field gradient of about 100 Tesla/m transverse to the velocity of the atoms, what would be the approximate separation between the two beams? How easily noticeable is this separation?

3. *Detecting snoopers.* An exercise in cryptography and measurement theory. Suppose Alice and Bob try to establish a one-time pad key using the protocol described in class. Discuss how Alice and Bob can detect snoopers, as suggested in item #5 in the list in Sec 5.2 of the class meeting outline for Tuesday, Jan. 20 (#03).

(a) Suppose that Claire "listens in" on all of the electrons. She does this by picking random directions for an S-G apparatus she places between Alice and Bob. With this apparatus, she doesn't block the beams, but she does check (using light, for example), to see which way the electron passes through the apparatus. Then Bob sends a message after having received Alice's confirmation. Since Bob and Alice share their choice of axes publicly, Claire knows which directions they have agreed on. Claire can then read some of the bits in Alice's message to Bob. What fraction of the bits can Claire read?

(b) Bob and Alice can reduce the ability of Claire to read their message by sharing some more information. Suppose they each broadcast to each other $1/2$ of the bits in their key, chosen at random. Will they agree on these bits if Claire was not listening? Will they agree on these bits if Claire was listening? Explain your answer and think a bit about the implications for cryptography.