Revision Sheet - PHY312

General Relativity

- Einstein's theory of GR rests on two principles:
  1. Principle of Equivalence - there is no way to distinguish *locally* between an inertial frame and a free falling frame of reference.
  2. Principle of General Coordinate Invariance - an observer in an arbitrary frame of reference should be able to discover the same laws of physics - all frames are equivalent.
- What do we mean by *locally* ? If we do experiments of a limited precision over a small region of spacetime we cannot distinguish a free falling frame over a truly inertial frame i.e we do not know whether a gravitational field is present. However, for a large enough spacetime region we will be able to infer the presence of gravity via its *tidal gravitational effects* - that is, two initially separated particles viewed from my freely falling frame will appear to approach each other.
- Einstein pictured tidal gravity as arising from underlying spacetime curvature.
- Particles falling freely in a gravitational field are pictured as following *geodesics* in the curved spacetime.
- A geodesic path between two points is the analog of the straightest possible path between the points and corresponds to the maximum possible *proper time* (as in special relativity).
- Einstein's *field equations* relate the density of spacetime curvature at some point in spacetime to the density of energy and mass at that point.
- Curved spacetime specified by giving a rule to compute (spacetime) distances at every point - a *metric.*

Motion in Schwarzschild spacetime

- The formula for the *Schwarzschild* metric is

$$\Delta s^2 = c^2 A(r)\Delta t^2 - \frac{\Delta r^2}{A(r)} - r^2\Delta\theta^2 \tag{1}$$

  This describes a static, spherically symmetric source of gravitation with $A(r) = 1 - \frac{2GM}{c^2 r}$. The event horizon $r_S = 2GM/c^2$.

- Know the formulae for the radial distance a shell observer (one static with respect to the black hole) would measure in terms of far away or r-coordinate r. And also the time between 2 events as measured on his watch given the time measured by a far-away observer t.

$$\Delta r_{shell} = \frac{\Delta r}{\sqrt{A(r)}} \tag{2}$$

$$\Delta t_{shell} = \Delta t\sqrt{A(r)} \tag{3}$$

- The energy E for a particle moving in this geometry is conserved.

$$E = mc^2\left(1 - \frac{r_S}{r}\right)\frac{\Delta t}{\Delta\tau} \tag{4}$$

- For a particle released from rest the expression for the speed measured by a far away observer is

$$\frac{\Delta r}{\Delta t} = -c\sqrt{\frac{r_S}{r}}A(r) \tag{5}$$

- The speed measured by a shell observer is given by

$$\frac{\Delta r_{shell}}{\Delta t_{shell}} = -c\sqrt{\frac{r_S}{r}} \tag{6}$$

- Know the formula giving the Schwarschild speed for an object released from rest at some r-coordinate r_o and falling radially inward

$$\frac{dr}{dt} = -cA(r)\left[\frac{r_S/r - r_S/r_O}{1 - r_S/r_O}\right]^{1/2} \tag{7}$$

- For general orbital motion we have another conserved quantity - the angular momentum L

$$L = mr^2\frac{\Delta\theta}{\Delta\tau} \tag{8}$$

- Know the form of the effective potential for material particles in orbit around a black hole. Understand the significance of drawing horizontal lines of constant energy. Know which points of the picture correspond to stable circular orbits, which to elliptical orbits and which yield the possibility of capture.
- Know how to compute L/m and E/mc^2 for an object projected from some fixed r-coordinate at speed v_{shell}.

Cosmology

- Under assumptions (experimental evidence favors this) of homogeneity and isotropy (know what those terms mean) the spacetime of the Universe looks like a set of 3d spaces evolving in a cosmic time t.
- These spaces may be positive curvature (sphere), flat, or negative curvature (hyperboloid).
- If we look at the solutions of Einstein's equations for this type of geometry we find solutions in which the size of the Universe is changing with time. In fact it seems as if the Universe was of zero size a finite time ago - the Big Bang.
- Shortly after the Big Bang the Universe was very hot and it subsequently cooled as the Universe expanded.
- The evidence for this comes from the cosmic microwave background, the abundances of light elements (nucleosynthesis) and Hubble's law.
- Hubble's law: two galaxies separate with a speed proportional to their distance apart.
- The standard Big Bang cosmology has several problems – most prominent of which is the observation that the observed Universe is much more homogeneous than one would have expected. These problems can be solved by assuming a period of very rapid expansion in the early Universe – **inflation**.

- The question of what will happen to the Universe in the future is determined by the matter density and pressure, the value of the cosmological constant and the sign of the spatial curvature.
- Modern observations favor a flat spatial geometry and a non-zero cosmic acceleration.