

## Homework 7

### Problem 1

(a) For a photon emitted outside the Schwarzschild radius and received at infinity we have

$$\lambda_{final} = \frac{\lambda_{initial}}{\sqrt{1 - \frac{2GM}{c^2 r_i}}} = \frac{4 \cdot 10^{-7}}{\sqrt{1 - \frac{2(6.67 \cdot 10^{-11})(2 \cdot 10^{30})}{(3.0 \cdot 10^8)^2 (5 \cdot 10^{30})}}} = 6.3 \cdot 10^{-7} m = 630 nm. \quad (1)$$

which would appear orange/red to the observer.

### Problem 2

(a)

$$\rho = \frac{Mass}{Volume} \quad (2)$$

so we have

$$M = \rho_{ave} \frac{4\pi}{3} r_s^3. \quad (3)$$

(b)

$$r_s = \frac{2GM}{c^2} \quad (4)$$

so

$$M = \frac{r_s c^2}{2G}. \quad (5)$$

(c) Setting the result from b and c equal we have

$$\frac{r_s c^2}{2G} = \rho_{ave} \frac{4\pi}{3} r_s^3 \quad (6)$$

rearranging and solving for  $r_s$  we have

$$r_s = \sqrt{\frac{3c^2}{8G\pi\rho_{ave}}} \quad (7)$$

(d) For  $\rho_{ave} = 1 \frac{kg}{m^3}$  we have

$$r_s = \sqrt{\frac{3(3.0 \cdot 10^8)^2}{8(6.67 \cdot 10^{-11})\pi(1)}} = 1.3 \cdot 10^{13} m \quad (8)$$

and so

$$\frac{r_s}{R_{pluto}} = \frac{1.3 \cdot 10^{13}}{6 \cdot 10^{12}} = 2.1. \quad (9)$$

(e)

$$\frac{M_{tot}}{M_{solar}} = \frac{r_s c^2}{2GM_{solar}} = \frac{(1.3 \cdot 10^{13})(3.0 \cdot 10^8)^2}{2(6.67 \cdot 10^{-11})(2 \cdot 10^{30})} = 4.4 \cdot 10^9 \quad (10)$$

### Problem 3

First of all we have

$$F_{tidal} = \frac{2GM}{r^3} \Delta r \quad (11)$$

and for

$$r = r_s = \frac{2GM}{c^2} \quad (12)$$

thus

$$F_{tidal} = \frac{2GM}{\left(\frac{2GM}{c^2}\right)^3} \Delta r = \frac{c^6}{4G^2 M^2} \Delta r. \quad (13)$$

(a)

$$F_{tidal} = \frac{(3.0 \cdot 10^8)^6}{4(6.67 \cdot 10^{-11})^2 (5 \cdot 2 \cdot 10^{30})^2} (1) = 4.1 \cdot 10^8 N \quad (14)$$

(b)

$$F_{tidal} = \frac{(3.0 \cdot 10^8)^6}{4(6.67 \cdot 10^{-11})^2 ((4.4 \cdot 10^9) \cdot 2 \cdot 10^{30})^2} (1) = 4.1 \cdot 10^8 N \quad (15)$$