Homework 4

Below I frequent put to use the energy relation

$$E^2 = p^2c^2 + m^2c^4.$$

where p is the magnitude of relativistic momentum. This relation holds for both an individual particle and a systems of particles.

Problem 1

(a)

$$m_A = m_B = 0$$

(b)

$$E_{A+B} = E_A + E_B = 3.0 \cdot 10^{-19}\text{J}$$

(c)

$$p_{A+B} = p_A + p_B = 10^{-27}\text{kg m/s}$$

(d) Solving for m in the energy relation we obtain

$$m_{A+B} = \frac{1}{c}\sqrt{\frac{E_{A+B}^2}{c^2} - p_{A+B}^2} = 0.$$

(e) Now we'll have:

$$E_{A+B} = E_A + E_B = 3.0 \cdot 10^{-19}\text{J}$$

$$p_{A+B} = p_A + p_B = -3.3 \cdot 10^{-28}\text{kg m/s}$$

Solving for m in the energy relation we obtain

$$m_{A+B} = \frac{1}{c}\sqrt{\frac{E_{A+B}^2}{c^2} - p_{A+B}^2} = 3.1 \cdot 10^{-36}\text{kg}.$$

(f) Now we'll have:

$$E_{A+B} = E_A + E_B = E_A + m_Bc^2 = 8.2 \cdot 10^{-14}\text{J}$$

$$p_{A+B} = p_A + p_B = 3.3 \cdot 10^{-28}\text{kg m/s}$$

Solving for m in the energy relation we obtain

$$m_{A+B} = \frac{1}{c}\sqrt{\frac{E_{A+B}^2}{c^2} - p_{A+B}^2} = 9.1 \cdot 10^{-31}\text{kg}.$$

Problem 2

(a)

$$E_{\text{tot}} = E_{e^-} + E_{e^+} = 2\sqrt{m_{e^-}^2 c^4 + p_{e^-}^2 c^2}$$

(b)

$$p_{tot} = p_{e^-} + p_{e^+} = p_{e^-} - p_{e^-} = 0$$

(c)

$$m_{tot} = \frac{1}{c}\sqrt{\frac{E_{tot}^2}{c^2} - p_{tot}^2} = \frac{E_{tot}}{c^2} = \frac{2\sqrt{m_{e^-}^2 c^4 + p_{e^-}^2 c^2}}{c^2} = 2\sqrt{m_{e^-}^2 + \frac{p_{e^-}^2}{c^2}}$$

(d) Energy is conserved so (where γ subscripts refer to photons)

$$E_i = E_f = E_{\gamma,1} + E_{\gamma,2} = 2E_\gamma$$

so we have

$$E_\gamma = \frac{1}{2}E_i = \sqrt{m_{e^-}^2 c^4 + p_{e^-}^2 c^2}.$$

(e) For a massless particle we have

$$E = pc$$

thus

$$p_\gamma = \frac{E_\gamma}{c} = \sqrt{m_{e^-}^2 c^2 + p_{e^-}^2}.$$

Problem 3

(a) Energy is conserved so we have

$$E_i = E_f \rightarrow E_{\gamma,i} + E_{e^-,i} = E_{\gamma,f} + E_{e^-,f}$$

or

$$E + mc^2 = E' + \sqrt{m^2c^4 + p^2c^2}. \tag{1}$$

And momentum is conserved so we have

$$p_i = p_f \rightarrow p_{\gamma,i} + p_{e^-,i} = p_{\gamma,f} + p_{e^-,f}$$

or

$$\frac{E}{c} = -\frac{E'}{c} + p$$

thus

$$E = pc - E'. \tag{2}$$

(b) Combing equations (1) and (2) and eliminating E' we get

$$E + mc^2 = pc - E + \sqrt{m^2c^4 + p^2c^2}$$

or

$$2E + mc^2 - pc = +\sqrt{m^2c^4 + p^2c^2}.$$

Squaring the above relation and with some cancelations we get

$$4E^2 + 4Emc^2 - 4Epc - 2mpc^3 = 0.$$

Solving for p yields

$$p = \frac{E + mc^2}{c\left(1 + \frac{mc^2}{2E}\right)}.$$

(c) Evaluating the above expression at $E = 2m$ yields

$$p = \frac{12mc}{5}.$$