Homework 3

Problem 1

$$p = m\gamma v$$
$$= \frac{m\frac{\Delta x}{\Delta t}}{\sqrt{1-\frac{\Delta x^2}{c^2 \Delta t^2}}}$$
$$= 1.75 \cdot 10^9 \text{kg m/s}$$

$$E = \sqrt{p^2c^2 + m^2c^4}$$
$$= 5.90 \cdot 10^{17}\text{J}$$
$$E_{\text{rest}} = mc^2 = 2.7 \cdot 10^{17}\text{J}$$
$$KE_{relativistic} = E - E_{\text{rest}} = 3.2 \cdot 10^{17} J$$
$$KE_{\text{Newton, non-relativistic}} = \frac{1}{2}mv^2 = 1.07 \cdot 10^{17}\text{J}$$

Problem 2

$$p_{\text{lab}} = m\gamma_{\text{lab}} v_{\text{lab}}$$
$$= \frac{mv_{\text{lab}}}{\sqrt{1-\frac{v_{\text{lab}}^2}{c^2}}}$$
$$= 3.5 \cdot 10^{-18}\text{kgm/s}$$

$$E_{\text{lab}} = \sqrt{p_{\text{lab}}^2 c^2 + m^2 c^4}$$
$$1.06 \cdot 10^{-9}\text{J}$$

To find quantities in the rocket frame we'll have to perform a Lorentz transformation:

$$p_{\text{rocket}} = \gamma \left(p_{\text{lab}} - \frac{vE_{\text{lab}}}{c^2} \right)$$
$$2.0 \cdot 10^{-18}\text{kg m/s}$$
$$E_{\text{rocket}} = \gamma \left(E_{\text{lab}} - vp_{\text{lab}} \right)$$
$$6.18 \cdot 10^{-10}\text{J}$$

Problem 3

(a)

$$E_A = E_{\text{A, rest}} = m_A c^2 = 20mc^2$$

(b) By conservation of energy we have

$$E_A = E_C + E_D$$

thus

$$E_D = E_A - E_C = 20mc^2 - 5mc^2 = 15mc^2$$

(c) Using the relation

$$E^2 = p^2 + m^2c^4$$

and rearranging we obtain

$$p_C = \frac{1}{c}\sqrt{E^2 - m^2c^4}$$

$$= \sqrt{21}mc.$$

(d) Using conservation of momentum,

$$p_{\text{intial}} = p_{\text{final}} = p_C + p_D = 0.$$

Thus

$$p_D = \sqrt{21}mc, \text{ oppositely directed from } p_C.$$

(e)Again using the energy relation and this time solving for the mass we obtain

$$m_D = \frac{1}{c}\sqrt{\frac{E_D^2}{c^2} - p_D^2}$$

Plugging in the relevant numbers we get

$$m_D = 2\sqrt{51}m = 14.3m.$$

(f) Lets first look at

$$m_C + m_D = 2m + 14.3m = 16.3m < 20m$$

so mass isn't conserved. This is consistent because mass in general is not conserved- its total energy that is conserved and some of the mass of the initial particle went into the kinetic energies of the subsequent particles. In this sense mass is on the same footing as any other form of energy, just like how potential energy can become kinetic energy.

Problem 4

(a) First we need the total power that hits the earth, this is (where I is intensity, or power divided by area)

$$P_E = IA_{\text{cross section of the earth}} = I\pi R_E^2 = 1.77 \cdot 10^{17} W.$$

So the mass converted per unit time in the Sun to supply the Earth is

$$\frac{\Delta m_E}{\Delta t} = \frac{\Delta E_E/c^2}{\Delta t} = \frac{1}{c^2}\frac{\Delta E_E}{\Delta t} = \frac{P_E}{c^2} = 1.96\text{kg/s}.$$

(b) Similarly,

$$P_{\text{total}} = IA_{\text{sphere with radius of earth's orbit}} = I4\pi R_E^2 = 3.88 \cdot 10^{26}\text{W}$$

$$\frac{\Delta m_{\text{total}}}{\Delta t} = \frac{P_{\text{total}}}{c^2} = 4.3 \cdot 10^9\text{kg/s}$$

(c) Below I put magnitudes on everything and extract the signs, i.e. $-5 = -|5|$. The total loss of mass in the Sun is amount lost to burning hydrogen plus the mass gained back in the form of formed Helium or

$$-\left|\frac{\Delta m_{\text{tot}}}{\Delta t}\right| = -\left|\frac{\Delta M_{\text{hyd}}}{\Delta t}\right| + \left|\frac{\Delta M_{\text{hel}}}{\Delta t}\right| \tag{1}$$

where ΔM_{i} is the total change the amount of ith mass flavor or

$$-\left|\frac{\Delta m_{\text{tot}}}{\Delta t}\right| = -\left|\frac{\Delta N_{\text{hyd}}}{\Delta t}\right| m_{\text{hyd}} + \left|\frac{\Delta N_{\text{hel}}}{\Delta t}\right| m_{\text{hel}} \tag{2}$$

where ΔN_i is the change in number of ith mass flavor. Now there is a $4:1$ hydrogen to helium ratio so can we rewrite equation (2) as

$$-\left|\frac{\Delta m_{\text{tot}}}{\Delta t}\right| = -\left|\frac{\Delta N_{\text{hyd}}}{\Delta t}\right| m_{\text{hyd}} + \frac{1}{4}\left|\frac{\Delta N_{\text{hyd}}}{\Delta t}\right| m_{\text{hel}}. \tag{3}$$

Solving for the hydrogen rate in equation (3) we obtain

$$\left|\frac{\Delta N_{\text{hyd}}}{\Delta t}\right| = \frac{\left|\frac{\Delta m_{\text{tot}}}{\Delta t}\right|}{m_{\text{hyd}} - \frac{m_{\text{hel}}}{4}}. \tag{4}$$

Rewriting and solving for hydrogen mass loss in equation (1) we obtain

$$\left|\frac{\Delta M_{\text{hyd}}}{\Delta t}\right| = \left|\frac{\Delta m_{\text{tot}}}{\Delta t}\right| + \frac{1}{4}\left|\frac{\Delta N_{\text{hyd}}}{\Delta t}\right| m_{\text{hel}}. \tag{5}$$

Inserting (4) into (5) we obtain

$$\begin{aligned}\left|\frac{\Delta M_{\text{hyd}}}{\Delta t}\right| &= \left|\frac{\Delta m_{\text{tot}}}{\Delta t}\right| + \frac{1}{4}\frac{\left|\frac{\Delta m_{\text{tot}}}{\Delta t}\right|}{m_{\text{hyd}} - \frac{m_{\text{hel}}}{4}} m_{\text{hel}} \\ &= \frac{\left|\frac{\Delta m_{\text{tot}}}{\Delta t}\right|}{1 - \frac{m_{\text{hel}}}{4m_{\text{hyd}}}} \\ &= 6.64 \cdot 10^{11} \text{kg/s}.\end{aligned}$$

(d) If we start with all hydrogen and end with all helium the Sun will burn

$$T = \frac{M_{\text{Sun}}}{\left|\frac{\Delta M_{\text{hyd}}}{\Delta t}\right|} = 3.1 \cdot 10^{18} s.$$