

Homework 2

Problem 1

'E' subscripts refer to earth's reference frame and 'A' denote the astronaut's reference frame. Working in the earth's reference frame

$$v_E = \frac{\Delta x_E}{\Delta t_E},$$

and time dilation relates the earth's time to the given astronaut's time

$$\Delta t_E = \Delta t_s \frac{1}{\sqrt{1 - \frac{v_E^2}{c^2}}}.$$

Combining the above equations and squaring yields

$$v_E^2 = \left(\frac{\Delta x_E}{\Delta t_E} \right)^2 \left(1 - \frac{v_E^2}{c^2} \right)$$

and solving for v_E we obtain

$$v_E = \frac{1}{\sqrt{\frac{1}{c^2} + \frac{\Delta t_A^2}{\Delta x_E^2}}}.$$

Plugging in the numbers this is simply c to a reasonable number of significant figures. Most calculators won't even give you any deviation from c for these numbers, so if you want to see how your result deviates from c , you will have to use some expansion. A useful one to remember for $\epsilon \ll 1$ is

$$(1 + \epsilon)^n \approx 1 + n\epsilon.$$

Now that we have the velocity we can just use the time dilation formula,

$$\Delta t_E = \Delta t_s \frac{1}{\sqrt{1 - \frac{v_E^2}{c^2}}} = 2.0 \cdot 10^6 \text{yr}.$$

Problem 2

We are given that the coordinates (x, t) in frame I and the coordinates (x', t') in frame II and $v_{\text{II wrt I}} = \frac{c}{2}$. The Lorentz transformations are

$$t_{\text{II}} = \gamma \left(t_{\text{I}} - \frac{vx_{\text{I}}}{c^2} \right)$$

$$x_{\text{II}} = \gamma (x_{\text{I}} - vt_{\text{I}})$$

where you should take note that v here is a component of a vector so it can be negative! If you get confused by the relative signs in Lorentz transformation just remind yourself that the relative signs inside the parenthesis for the x_{II} transformation should make sense in the nonrelativistic limit ($\gamma \rightarrow 1$). Now we have

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2}{\sqrt{3}}.$$

Plugging in numbers we have

$$t_{\text{II}} = \frac{2}{\sqrt{3}} \left(t_{\text{I}} + \frac{x_{\text{I}}}{2c} \right)$$

$$x_{\text{II}} = \frac{2}{\sqrt{3}} \left(x_{\text{I}} + \frac{ct_{\text{I}}}{2} \right)$$

which makes sense since if II is moving to the left of I we should have $x_{\text{II}} > x_{\text{I}}$ even in the nonrelativistic limit.

Problem 3

Taking $(t_{\text{II}}, x_{\text{II}}) = (1.0 \cdot 10^{-8}\text{s}, 2\text{m})$ and using relations from problem 2 we get

$$t_{\text{I}} = 7.7 \cdot 10^{-9}\text{s}$$

$$x_{\text{I}} = .58\text{m}.$$

Problem 4 Using the relativistic velocity addition formula

$$v = \frac{v' + v_{\text{rel}}}{1 + \frac{v'v_{\text{rel}}}{c^2}}$$

and as always taking care that the v 's that appear are components of vectors that can be negative we get

$$v = \frac{v_{\text{II}} + v_{\text{rel}}}{1 + \frac{v_{\text{II}}v_{\text{rel}}}{c^2}}$$

$$= \frac{\frac{c}{2} - \frac{c}{2}}{1 - \frac{1}{4}} = 0.$$

Problem 5 Using the velocity addition relation again we have

$$\begin{aligned} v &= \frac{c + v_{\text{rel}}}{1 + \frac{cv_{\text{rel}}}{c^2}} \\ &= \frac{c + v_{\text{rel}}}{\left(\frac{1}{c}\right)(c + v_{\text{rel}})} \\ &= c. \end{aligned}$$

Note that that this is true regardless of what v_{rel} is.

Problem 6 This question refers to the spacetime interval

$$\Delta s^2 = c^2 \Delta t^2 - \Delta x^2.$$

A spacelike interval is such that

$$\Delta x^2 > c^2 \Delta t^2.$$

A timelike interval is such that

$$\Delta x^2 < c^2 \Delta t^2.$$

If two events are casually linked their spacetime interval must be timelike. I can't choose a frame of reference in which a timelike interval becomes spacelike- the spacetime interval is invariant so it certainly can't change by a sign!