PHY312 - lecture 9

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Recap

- Saw that free fall frames serve as good approximations to inertial frames.
- Indeed, effects of gravity can be almost eliminated by jumping to such a FFF.
- Relies on equality of gravitational and inertial masses
- Equivalent to old observation of Galileo - that all bodies fall equally fast under gravity independent of their mass, composition etc
- Einstein realized importance of this – elevated it to a principle

Principle of equivalence

There are no *local* experiments that can distinguish free fall in a gravitational field from uniform motion in the absence of a gravitational field

- Perhaps gravity is not a property of any body but a property of space(time) itself ...?
- What about the caveat local ?

General frames of reference

- Since FFF play a special role and these are accelerating one is motivated to formulate laws of physics so that they look same in *any* FOR (not just inertial).
- This is why the resultant theory is called General Relativity GR as opposeed to special relativity which describes only inertial frames.
- Have seen that such theory will necessarily involve gravity ....

Newtonian gravity

- **Newtonian Gravity.** Universal (attractive) force of gravity acts between *all* bodies.
- $F = -\frac{G_N m_1^G m_2^G}{r^2}$. G_N is Newton's constant $G_N = 6.67 \times 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2}$. m^G is the *gravitational mass*.
- Analogous to the electric charge in electromstatics – specifies the strength of the coupling between the particle and the gravitational (electric) field.
- Gravity is however a much weaker force then electromagnetism. For example, the ratio of gravitational to electric forces between two electrons $F_G/F_E \sim 10^{-22}$!)
- Succesful prediction of planetary orbits, tides etc

What’s wrong

Requires an instantaneous gravitational force - in conflict with special relativity which states that no physical disturbance or interaction can propagate with a speed greater than the speed of light!

Gravity vs acceleration

- What do I mean by locally ?
- Think about the elevator free falling above surface of Earth. Release two ball bearings 20m apart at each side. What happens ?
- Initially they will remain 20m apart and stationary in falling frame.
- But after 8 secs (315 m fall) they will be 1mm closer together !
- Why – the balls fall towards center of Earth on slowly converging trajectories ...
- This would not happen for balls released in an accelerating elevetor without gravity.
- Can distinguish gravity from uniform acceleration!

Tidal gravity

- Real gravitational fields vary in space and their effects cannot be eliminated by jumping to an accelerated FOR.
- In the context of the moon's gravitational pull on Earth these effects result in tides – the moon pulls more strongly on the side of Earth nearest to it and the oceans flow under the influence of this difference of forces.
- Hence "real" gravity is tidal gravity – can detect a difference because of tidal effects.
- But ... if I do the 2 ball bearing expt over shorter times or with a smaller initial separation the effect is smaller – and eventually undetectable – thus over local regions of spacetime we cannot distinguish gravity from pure acceleration.

More on tidal effects

- The magnitude of these tidal gravitational effects depends on the strength of the gravitational field and hence the mass of the gravitating body.
- As we saw they are also proportional to the size of the spacetime region. Hence they can always be made smaller than the resolution of my measuring apparatus by going to small regions.
- Bodies falling toward the center of a spherically symmetric object like the Earth (or a blackhole) are stretched along their line of motion and squashed transverse to it.

Consequences - light bending

- Light is bent by a gravitational field. Imagine firing a beam of photons from the frame of an accelerating elevator – they will follow a curved path (since can think of them as particles of small mass $m = E/c^2$). But by the principle of equivalence the same thing should hold in a gravitational field.
- Light grazing the Sun should be bent by an angle

$$\Delta\phi \sim \frac{GM_S}{R_S c^2}$$

See in experiments.

More ...

- Clocks are slowed in a gravitational field. Imagine placing a clock in an accelerating FOR. Time dilation (small time interval) says

$$d\tau = dt\left(1 - v(t)^2/c^2\right)^{\frac{1}{2}}$$

Integrating:

$$\tau = t\left(1 - \frac{T^2 a^2}{6c^2}\right)$$

But distance gone is $\Delta s = \frac{1}{2}aT^2$ so find

$$\frac{\tau}{t} \sim 1 - \frac{1}{3}\frac{a}{c}\frac{\Delta s}{c}$$

- But use POE to swap a for g.

continuing

- Time measured on a clock at rest in a gravitational field should run more slowly compared to one at infinity. For Earth subsitute radius R for Δs.

$$\frac{\tau}{t} \sim 1 - \frac{GM}{c^2}\frac{1}{R}$$

Gravitational redshift

- Consider photon emitted out from a spherical gravitational field with energy E. Has mass E/c^2 and hence a potential energy $-GM(E/c^2)/R$.
- Needs to lose this amount of kinetic energy to escape. Lowers its measured energy at infinity (for photons also have $E = hf$). Lower frequency, longer wavelength – redshift.

$$\frac{\Delta E}{E} = \frac{\Delta f}{f} \sim 1 - \frac{GM}{c^2}\frac{1}{R}$$

- Consistent with clock slowing – thinking of oscillations of wave as like a clock.

General relativity

- Einstein argued that all observers, whether inertial or not should be capable of discovering the correct laws of physics. Thus he proposed the following as a logical completion of special relativity
Principle of General Relativity: All observers are equivalent
- Einstein used the principle of equivalence, this principle of general relativity, together with the requirements that his theory be the simplest possible extension of Newton's ideas (and reduce to the same predictions for small velocities etc) to derive his theory of GR. From our discussions so far it is clear that such a theory has the ability to describe not just accelerated frames but gravity.