PHY312 - lecture 8

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Recap

- Energy-momentum conservation.
- Special relativity respects causality – if one observer sees event A causing event B, then all (inertial) observers in relative motion will agree that A causes B provided that A and B are separated by a timelike (or null) spacetime interval $\Delta s^2 \geq 0$
- Requires that all physical motions propagate at $v \leq c$.
- Spacetime partitioned into spacelike and timelike separated events by the light cone.

Inertial frames again

- In practice it is easy to find a inertial FOR in which electric, magnetic, nuclear forces are all negligible. But what about gravity ?
- One option: go to interstellar space away from all masses etc
- There is another way. In presence of gravity jump to a FOR which is falling freely under gravity a so-called free fall frame FFF.
- Quick thought experiment. Imagine an observer in a freely falling elevator. He throws a coin. What does he observe ?
- He will see coin move at constant velocity !
- Thus freely falling frames FFF serve as (almost) inertial frames even presence of gravity.

Newtonian analysis

- Denote coordinates of coin relative to Earth by x_{CE}, coordinates of freely falling elevator frame relative to Earth by x_{FE} and coordinates of coin relative to elevator by x_{CF}.
- We have: $x_{CF} = x_{CE} - x_{FE}$
- But from Newton's 2nd law:

$$m_F^I \frac{d^2 x_{FE}}{dt^2} = g m_F^G$$

$$m_C^I \frac{d^2 x_{CE}}{dt^2} = g m_C^G$$

Provided $m^I = m^G$

$$\frac{d^2 x_{CF}}{dt^2} = 0!$$

Conclusions

- Thus, while motion of coin would follow parabola in Earth frame it is uniform in FFF !
- Requires inertial mass=gravitational mass. It took Einstein to understand the significance of this apparently trivial statement.
- Can use the laws of special relativity within such FFF.
- The effects of gravity can be (almost) eliminated within such a frame ...

Accelerating elevator

- Consider now a rocket in empty space. Imagine accelerating the rocket.
- Throw the coin again. What will you see ?
- Relative to the rocket the trajectory of the coin will be curved.

$$\frac{d^2 x_{CF}}{dt^2} = -a$$

- This looks like equation of motion for coin near Earth's surface where acceleration due to gravity is a!
- Thus gravity can be mimicked by accelerating frames of reference.

Principle of equivalence

There are no *local* experiments that can distinguish free fall in a gravitational field from uniform motion in the absence of a gravitational field

- Requires equality of gravitational and inertial mass.
- Follows from the equal gravitational acceleration of all bodies independent of their mass (Galileo ..)
- Perhaps gravity is not a property of any body but a property of space(time) itself ...?
- What about the caveat local ?

General frames of reference

- Since FFF play a special role and these are accelerating one is motivated to formulate laws of physics so that they look same in *any* FOR (not just inertial).
- This is why the resultant theory is called General Relativity GR as opposeed to special relativity which describes only inertial frames.
- Have seen that such theory will necessarily involve gravity ....

Newtonian gravity

- **Newtonian Gravity**. Universal (attractive) force of gravity acts between *all* bodies.
- $F = -\frac{G_N m_1^G m_2^G}{r^2}$. G_N is Newton's constant $G_N = 6.67 \times 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2}$. m^G is the *gravitational mass*.
- Analogous to the electric charge in electromstatics – specifies the strength of the coupling between the particle and the gravitational (electric) field.
- Gravity is however a much weaker force then electromagnetism. For example, the ratio of gravitational to electric forces between two electrons $F_G/F_E \sim 10^{-22}$!)
- Succesful prediction of planetary orbits, tides etc

What's wrong

Requires an instantaneous gravitational force - in conflict with special relativity which states that no physical disturbance or interaction can propagate with a speed greater than the speed of light!

Gravity vs acceleration

- What do I mean by locally ?
- Think about the elevator free falling above surface of Earth. Release two ball bearings 20m apart at each side. What happens ?
- Initially they will remain 20m apart and stationary in falling frame.
- But after 8 secs (315 m fall) they will be 1mm closer together !
- Why – the balls fall towards center of Earth on slowly converging trajectories ...
- This would not happen for balls released in an accelerating elevetor without gravity.
- Can distinguish gravity from uniform acceleration!

Tidal gravity

- Real gravitational fields vary in space and their effects cannot be eliminated by jumping to an accelerated FOR.
- In the context of the moon's gravitational pull on Earth these effects result in tides – the moon pulls more strongly on the side of Earth nearest to it and the oceans flow under the influence of this difference of forces.
- Hence "real" gravity is tidal gravity – can detect a difference because of tidal effects.
- But ... if I do the 2 ball bearing expt over shorter times or with a smaller initial separation the effect is smaller – and eventually undetectable – thus over local regions of spacetime we cannot distinguish gravity from pure acceleration.