PHY312 - lecture 7

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Review

- energy-momentum vector $\mathcal{P} = (E/c, p_x, p_y, p_z)$
- Contains relavistic energy, momentum. Invariant length is rest mass.
- Conserved.

Example

- A photon moving with energy E collides with a stationary atom with (rest) mass m. The photon is absorbed and the recoils. Work out formulae for:
- The mass of the atom after collision
- The momentum of the atom after the collision.
- How fast it is traveling after collision (as viewed from the original FOR at which it was at rest)
- A typical visible light photon carries approx 1×10^{-18}J of energy while a hydrogen atom has mass approx 10^{-26} kg. What is the velocity of the recoiling H atom ?

Solution

- Conservation of energy and momentum:

$$\begin{aligned} E + mc^2 &= E_f \\ p &= E/c = p_f \end{aligned}$$

- Total mass $M^2c^2 = E_f^2/c^2 - p_f^2 = 2Em + m^2c^2$
- To find velocity equate $Mv\gamma = E/c$. Find:

$$\frac{v}{c} = \frac{1}{1 + mc^2/E}$$

- Putting $E = 10^{-18}$, $m = 10^{-26}$, $c = 3 \times 10^8$ find $\frac{v}{c} \sim 10^{-8}$ ie few m/s.

Photon rockets

- Perfect rocket engine combines matter and antimatter to create photons directed backwards.
- Initial mass M. After all fuel is burnt moves with speed v and has mass fM where $f < 1$
- What is v as function of f?
- Conservation of $\mathcal{P}$ needed !

Solution

- Consider before fuel burning and after all fuel burnt. Apply conservation of momentum and energy.

$$\begin{aligned} Mc^2 &= E_{rad} + \gamma M f c^2 \\ E_{rad}/c &= p \end{aligned}$$

  where $p^2 = (\gamma M f c)^2 - M^2 f^2 c^2$ (from invariant length of final rocket E-P vector)
- Find:

$$f^2 - 2f\gamma + 1 = 0$$

Consequences

- $f = \gamma - \sqrt{\gamma^2 - 1}$
- What f for $\gamma = 10$? What for $\gamma = 100$?

Recap causality

- Said 2 events that are spacelike separated cannot be causally related (used argument that no physical signal can propagate faster than speed of light).
- Here, give another more direct argument. Consider 2 events A $(0,0)$ and B (x,t) and assume A causes B.
- In some FOR $t > 0$. B happens later.
- Question: can one jump to another FOR with coords (x',t') where $t' < 0$. i.e in such frame B occurs earlier!
- Lorentz transformation implies:

$$ct' - \frac{v}{c}x' < 0$$

- Compatible with $s^2 = c^2t'^2 - x'^2$? Since $v/c < 1$ we are safe for timelike intervals only ...

Inertial frames again

- In practice it is easy to find a inertial FOR in which electric, magnetic, nuclear forces are all negligible. But what about gravity ?
- One option: go to interstellar space away from all masses etc
- There is another way. In presence of gravity jump to a FOR which is falling freely under gravity ...
- Quick thought experiment. Imagine an observer in a freely falling elevator. He throws a coin. What does he observe ?
- He will see coin move at constant velocity !
- Thus freely falling frames FFF serve as (almost) inertial frames even presence of gravity.

Newtonian analysis

- Denote coordinates of coin relative to Earth by x_{CE}, coordinates of freely falling elevator frame relative to Earth by x_{FE} and coordinates of coin relative to elevator by x_{CF}.
- We have: $x_{CF} = x_{CE} - x_{FE}$
- But from Newton's 2nd law:

$$\begin{aligned} m_F^I \frac{d^2 x_{FE}}{dt^2} &= g m_F^G \\ m_C^I \frac{d^2 x_{CE}}{dt^2} &= g m_C^G \end{aligned}$$

Provided $m^I = m^G$

$$\frac{d^2 x_{CF}}{dt^2} = 0!$$

Conclusions

- Thus, while motion of coin would follow parabola in Earth frame it is uniform in FFF !
- Requires inertial mass=gravitational mass. It took Einstein to understand the significance of this ...
- Can use the laws of special relativity within such FFF.
- The effects of gravity can be (almost) eliminated within such a frame ...

Accelerating elevator

- Consider now a rocket in empty space. Imagine accelerating the rocket.
- Throw the coin again. What will you see ?
- Relative to the rocket the trajectory of the coin will be curved.

$$\frac{d^2 x_{CF}}{dt^2} = -a$$

- This looks like equation of motion for coin near Earth's surface where acceleration due to gravity is a!
- Thus gravity can be mimicked by accelerating frames of reference.

Principle of equivalence

There are no *local* experiments that can distinguish free fall in a gravitational field from uniform motion in the absence of a gravitational field

- Requires equality of gravitational and inertial mass.
- Follows from the equal gravitational acceleration of all bodies independent of their mass (Galileo ..)
- What about the caveat local ?