PHY312 - lecture 6

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Review

- Vectors in spacetime: sets of 4 numbers (a_t, a_x, a_y, a_z) which transform like spacetime coordinates under LT. Length squared $a_t^2 - a_x^2 - a_y^2 - a_z^2$ same for all inertial observers.
- Important example: energy-momentum vector $\mathcal{P} = (E/c, p_x, p_y, p_z)$
- All laws of physics written in terms of such vectors: ensures satisfy relativity principle.

Energy-momentum vector

- Special relativity unites time with space but also energy with momentum and mass with energy !
- $E/c = m_0 c\gamma$. Hence $E \rightarrow \infty$ as $v \rightarrow c$. Physical reason why cannot go faster than speed of light.
- $p = m_0 v\gamma$. Momentum also infinite as $v \rightarrow c$. Notice relativistic momentum exceeds Newtonian expression since $\gamma \geq 1$.
- What is mass ? Energy measured in FOR in which particle is at rest ...

Examples

- Eg. A ball of mass 1 kg is travelling in the x-direction with speed 1 m/s. According to Einstein what is its energy and what is its momentum ? What values would Newton ascribe to the ball ?
- Now consider a proton of mass 1.67×10^{-27} kg moving at 0.9c in the x-direction. What is its relativistic energy and momentum ? What would be its energy and momentum according to Newton ?

More examples

- The Fermilab Tevatron is the largest US accelerator for doing high energy physics. In it protons and antiprotons are accelerated to speeds close to the speed of light and made to collide. The protons in the beam have energies of 900 Gev (1 Gev is 1.6×10^{-10} Joules). The proton rest mass is 1.67×10^{-27} kgs. What is the ratio of the speed of the particles to the speed of light ? (take $c = 3.0 \times 10^{8} m/s$).
- The protons and antiprotons are accelerated around a circlar ring of circumference approx 4 miles. How long (in the rest frame of the accelerator) does it take a proton/antiproton to go once around the ring ? How long does one of the protons think it takes .. ?

Many particles

- For system of particles get the total energy-momentum vector by adding up the energies and momenta for each.

$$P_{\text{tot}} = \left(\sum_i E_i/c, \sum_i p_i \right)$$

- What is mass of system ? Length of total energy momentum vector. In general this is not the sum of all rest masses !
- i.e the total length of a vector is not in general the sum of a single component of each vector!
- This is the price one pays for unifying mass energy with momentum. One loses the notion of absolute mass that all observers agree on.

Examples

- Consider 2 particles both of (rest) mass m, one at rest the other moving with total energy $4mc^2$. What is
  1. Total energy of the system
  2. Total momentum of the system
  3. Total mass of the system
- ans: $5mc^2$, $\sqrt{15}mc$, $\sqrt{10}m$

More

- Photons are massless particles. Hence $m^2c^2 = 0 = E^2/c^2 - p^2$. i.e $E = pc$.
- Consider two photons - initially travelling in same direction with energies $E = 3c$ and $E = 1c$.
  1. What is their total energy ?
  2. Hence what is their total momentum ?
  3. What is the mass of the 2 photons ?
  4. What happens when photons headed in opposite directions ?
- ans: $4c$, 4, 0, $\sqrt{12}/c$

Moral

- Mass has different interpretation in relativity than Newtonian theory. Mass becomes rest mass – length of particles energy-momentum vector.
- Total mass of system is just length of total energy-momentum vector of system.
- In rest frame of particle proportional to energy via $E = m_0 c^2$.
- Energy and momentum are united into more fundamental object – energy-momentum vector $\mathcal{P}$.

Conservation of $\mathcal{P}$

- In Newtonian mechanics E and p conserved separately. In relativity simply have conservation of $\mathcal{P}$. Valid in any inertial FOR.
- Length of $\mathcal{P}$ conserved – conservation of mass also !

Example

- A photon moving with energy E collides with a stationary atom with (rest) mass m. The photon is absorbed and the recoils. Work out formulae for:
- The mass of the atom after collision
- The momentum of the atom after the collision.
- How fast it is traveling after collision (as viewed from the original FOR at which it was at rest)
- A typical visible light photon carries approx 1×10^{-18}J of energy while a hydrogen atom has mass approx 10^{-26} kg. What is the velocity of the recoiling H atom ?

Solution

- Conservation of energy and momentum:

$$\begin{aligned} E + mc^2 &= E_f \\ p &= E/c = p_f \end{aligned}$$

- Total mass $M^2c^2 = E_f^2/c^2 - p_f^2 = 2Em + m^2c^2$
- To find velocity equate $Mv\gamma = E/c$. Find:

$$\frac{v}{c} = \frac{1}{1 + mc^2/E}$$

- Putting $E = 10^{-18}$, $m = 10^{-26}$, $c = 3 \times 10^8$ find $\frac{v}{c} \sim 10^{-8}$ ie few m/s.