PHY312 - lecture 4

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Review

- Events physical things. Can be represented by coordinates (x, t). These change when move from one FOR to another.
- All observers agree on distance between events as measured by $\Delta s = \sqrt{c^2 \Delta t^2 - \Delta x^2}$.
- If $\Delta s^2 > 0$ can define proper time $\tau = \Delta s / c$. For such timelike interval τ can be measured by clock that moves so that $\Delta x = 0$.
- Set of all events called spacetime. At given point divided into two regions by light cone.

Lorentz transformations I

- Imagine two frames (x, t) and (x', t') with relative velocity v. Imagine two events labeled $(0, 0)$ and (x, t) in one frame and $(0, 0)$ and (x', t') in second.
- From rocket experiment where $x' = 0$ we know:

$$t = t' \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x = vt = vt' \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Would like to know how to do this if $x' \neq 0$.
- The relationship between these two coordinate systems is called the Lorentz transformation.

Lorentz transformations II

- Assume general form

$$\begin{aligned} x &= \gamma\left(Ax' + vt'\right) \\ t &= \gamma\left(Bx' + t'\right)\end{aligned}$$

  where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

- Require $c^2t^2 - x^2 = c^2t'^2 - x'^2$.

Lorentz transformations III

- Find:

$$\begin{aligned} c^2\gamma^2\left(B^2-(A/c)^2\right) &= -1 \\ c^2B - Av &= 0 \end{aligned}$$

- Solving these leads to

$$\begin{aligned} t &= \gamma\left(t' + vx'/c^2\right) \\ x &= \gamma\left(x' + vt'\right) \end{aligned}$$

- For small velocities these reduce to $x = x' + vt'$ and $t = t'$ as for Newton!

Electricity and magnetism

- Maxwell's equations of EM take the same form in all FOR provided the coordinates are transformed according to the Lorentz transformations (LT).
- Just like $F = ma$ having same form in Newtonian mechanics provided $t' = t$, $x' = x + vt$ ...
- Thus special relativity completely in agreement with laws of EM

Addition of velocities

- Can use LT to derive how relative velocities work in relativity (remember – cannot be like in Newtonian mechanics ...)
- Divide LT equations for x, t to find:

$$v = \frac{v' + v_{rel}}{1 + v' v_{rel}/c^2}$$

  where $\frac{x}{t} = v$ and $\frac{x'}{t'} = v'$ and have replaced v by $v_{\rm rel}$.
- Notice reduce to usual ones as $c \to \infty$.

Some examples

- Suppose I measure a light beam to be traveling at speed c relative to some FOR. If I observe the light beam from another FOR traveling (in the positive x direction) with speed $v_{\rm rel}$ relative to first what speed would I observe?
  - According to Newton
  - According to Einstein
- I observe a rocket traveling at $4/5c$ in the positive x direction. A probe leaves the rocket in the positive x direction also at $4/5c$ *relative* to the rocket. How fast do I observe the probe to be going ?