PHY312 - lecture 3

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Review

- In Special Relativity there is no absolute (observer independent) notion of space and time separately – only a funny fusion of the two – the thing we have been calling spacetime.

  Hendeforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a union of the two will preserve an independent reality

  Herman Minkowski (1864-1909)

- All inertial observers agree on the distance in spacetime between 2 events $\Delta s^2 = c^2\Delta t^2 - \Delta x^2$

Proper time

- Define proper time between the two events $\tau = \Delta s/c$. Time interval measured by clock traveling with the rocket. Thus spacetime interval has a very physical interpretation.
- For the proper time to be a *real* number **the velocity must be less than c!**. Maximum velocity ..
- What is shape of this spacetime path for a free particle ?
- What is the length of the path for a photon ?

Timelike, spacelike, null

- Consider spacetime interval connecting 2 events. If $\Delta s^2 > 0$ interval is said to be timelike. Massive particles travel along worldlines which are timelike (implies their speed $v < c$, time measured in co-moving frame real). Two events which are timelike separated may influence each other (casually connected).
- $\Delta s^2 < 0$ also possible. Spacelike separated events. Such events are casually unconnected as no physical influence can propagate between them (would imply $v > c$). Can find a FOR where two events simultaneous.
- Events can be null separated $\Delta s = 0$. Eg. light (any massless particle)
- Causal structure of spacetime encoded in light cone.

Light cone

- Draw spacetime diagram plotting x horizontally (and maybe y into page, suppress z) and ct upwards.
- Consider an event occuring at origin. Rest of spacetime partitioned into 2 regions by the null (light) lines $x = \pm ct$. Past of event is set of points in lower cone. Future of event lies in upper cone. Boundaries of cone are worldlines of light rays which are emitted/received at event.
- Notice that we are not saying that all points in backward light cone influence the origin point – merely that they *may*. Conversely, *no* points outside light cone can influence or be influenced by the origin event.

Twin paradox - an exercise

- Consider two identical twins – twin A on Earth and another traveling away at speed v (twin B). Give an expression for the spacetime interval in Earth frame in terms of the space distance covered by rocket Δx_A and time as measured in Earth frame Δt_A ?
- What is Δx_B - the spatial distance covered in rocket frame?
- Hence write down an expression for Δt_B.
- Show that $\Delta t_B < \Delta t_A$. Thus "time for moving twin runs more slowly than for twin who stays home"
- Paradox. Use relativity to argue that B is at rest in rocket while A (and Earth!) hurtles off backwards at speed $-v$. I would conclude $\Delta t_A < \Delta t_B$. Contradiction ?

Resolution

- To see physical contradiction need to bring the “moving twin” back and compare.
- Draw/sketch a spacetime diagram containing the worldlines of both twins. Are the two worldlines equivalent ? (hint: use Earth FOR and imagine send twin B out at speed v and back at -v and ignore periods of acceleration/deceleration)

Straight line proper time

- From picture it should be clear that the “moving twin” (the one who has to undergo acceleration) has experienced a shorter proper time than his “stay-at-home” twin (unaccelerated). He has aged less.
- Imagine changing the velocity of trip. The more $v \rightarrow c$ the shorter his proper time (age).
- The straight worldline has the biggest proper time. It turns out that any non-straight worldline has a shorter proper time than a straight worldline between the 2 events!.

Principle of maximal ageing

- The straight line is the longest distance in spacetime between two events.
- Contrast this to ordinary space where straight lines are shortest way to get between 2 points.
- Principle of maximal ageing: Free particles move along worldlines of maximal proper time

Time dilation revisited

- Time separation ascribed by one observer to 2 events is less than the time interval measured for those same 2 events from a frame in which they occur at same spatial point. eg the rocket experiment: $\Delta t = \Delta t_R \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
- This is not a “physical slowing of moving clocks”. That phrase implicitly assumes a preferred, absolute time coordinate with respect to which the statement makes sense ... Correct interpretation is that time coordinates, like space coodinates, have no absolute meaning and will change when I change FOR. Only proper time τ has an invariant, absolute meaning which all observers agree on as it measures a distance in spacetime.

Length contraction

- Imagine a proton traveling at speed $3/4c$ between detectors 2 m apart At each detector a flash of light is emitted (event).
- What is the time interval between flashes as seen in lab?
- What is it in the frame of the proton ?
- Imagine proton is at rest and lab moves at speed $-3/4c$. Is the distance between the 2 events from proton frame smaller or larger than 2 m ?
- This is called length contraction and is the effect that moving objects appear to be contracted along the line of motion by the factor $\sqrt{1-\frac{v^2}{c^2}}$

Summary

- Time and space have no separate, absolute identity in relativity.
- Getting away from commonsense feeling of absolute time is the only real difficulty in understanding the theory.
- Absolute time is replaced by absolute proper time - distance in spacetime.
- Free particles move along worldlines in spacetime along which their proper time is maximal – they “age” the most.
- Spacetime is divided into past, present and future by the light cone.