

# Relativity and Cosmology

lecture 22

# Recap lecture 21

- Observed homogeneity and isotropy of Universe on large scales implies rather simple FRW metric. Only variable is scale factor  $a(t)$ . 3 spatial topologies - flat seems preferred.
- Same cosmological principle fixes energy momentum in terms of density and pressure with  $p = w\rho$ .
- Solutions of GR boil down to 2 equations for scale factor  $a(t)$ .
- Acceleration requires  $w < -1/3$  ( $w = -1$  is cosmological constant)
- Can solve equations for different  $w$  – AdS, dS or flat space.

# Big Bang

- Hubble's law implies expanding Universe. Seen that this compatible with Einstein's equations.
- Run time backward  $a(t) \rightarrow 0$  at some finite time in past – Universe would be small, highly curved and hot.
- At  $t = 0$  all of spacetime and matter appeared out of this singularity – Big Bang. Starts to expand according to GR.

# Timeline of events

- $t \sim 10^{-43}$  s. Regime of quantum gravity. Unknown in details but rather general arguments say that gravity becomes as strong as nuclear forces, and must be combined with quantum mechanics in some way. Horizon distances (distances light has travelled since  $t = 0$ ) are comparable to typical distances between collisions of particles! Universe is Planck scale

$$l_P = \sqrt{\frac{Gh}{c^3}} = 10^{-33}m. \text{ Continues to expand.}$$

- Exotic particle regime from  $t = 10^{-43}$  to  $t = 10^{-5}$  s Ultra relativistic matter. Details depend on your favourite “grand unified model”. Naively,  $a \sim \sqrt{t}$  (but see later discussion of inflation).
- Electron-positron production regime  $t = 10^{-5}$  to  $t = 1$ . Free quarks..

# Timeline continued

- Nucleosynthesis regime  $t \sim 100 - 1000s$ . Hydrogen, helium and the lightest elements are formed. Observed abundances agree.
- Decoupling time  $t = 100,000$  years. Neutral atoms form and photons propagate freely. This is the surface of last scattering revealed by observations of the microwave background CMB.
- $t > 100,000$  years. Pure expansion/galaxy formation. Scale factor grows like  $t^{2/3}$ . Except ...
- Today observe acceleration – cosmological constant ? Exponential expansion ?

# Explanation

- To understand these regimes we need to understand two concepts 1. Thermal equilibrium and 2. Particle Production Thresholds
- Thermal equilibrium. If large number of particles which collide and interact frequently (compared to expansion rate) system is randomized and total energy distributed in statistical way over the particles. Mean particle energy is  $k_B T$  where  $k_B = 1.38 \times 10^{-23} J/K$  and  $T = 1/a(t)$ .

$$a(t) = a_0 \left( \frac{t}{t_0} \right)^{2/3(1+w)}$$

- As we roll time backwards the Universe heats up. New physics kicks in whenever the average energy allows for production of new particles

# Particle Production thresholds

- As the energy of say a photon increases it becomes possible to create matter out of photon photon collisions via  $E = mc^2$ .
- This minimal energy will be reached when  $kT = mc^2$ .
- Above this temperature particles and antiparticles with mass  $m$  will be liberated.
- Conversely as the Universe expands and cools all electrons-positron pairs will combine into photons once temperature is low energy and Universe big enough.
- Other threshold temps occur when this thermal energy is equal to important other energy scales involved with atoms and nuclei eg. binding energy of electrons in atoms or the binding energies of simple nuclei (decoupling time and nucleosynthesis time)

# Whats wrong with this picture ?

- Flatness problem. We can rewrite the equation for the Hubble parameter in a slightly different form

$$\Omega - 1 = \frac{k}{H^2 a^2}$$

where  $\Omega = \frac{\rho}{\rho_c}$  and  $\rho_c = \frac{3H^2}{8\pi G}$ . So  $k = 0$  (flat) requires  $\Omega = 1$  and  $\rho = \rho_c$ . Notice though that  $\rho_c$  is not in general a constant.

- We can show that

$$\frac{d\Omega}{da} = (1 + 3w) \frac{\Omega(\Omega - 1)}{a}$$

# Flatness continued

- The problem is that if you require that  $\Omega$  is near 1 almost all cosmic time. But this in general is hard to achieve since  $\frac{d\Omega}{da}$  is not in general zero. Notice though that if  $\Omega = 1 + \epsilon$  at some point in the past the derivative looks like

$$\frac{d\Omega}{da} = (1 + 3w) \frac{\epsilon}{a}$$

Thus it only for  $1 + 3w < 0$  will it remain so – for ordinary matter it will flow away from the flat point very quickly.

Thus for regular matter it is an amazing coincidence that  $\Omega$  is observed to be close to unity. This is called the **flatness problem**.

- Notice though that cosmic acceleration would force this factor to be negative and render  $\Omega = 1$  “natural”.

# Horizon problem

- Horizon problem. Lets see how far a photon can propagate in time  $t$  from the Big Bang. It is given by

$$d_H = a(t)\Delta r = a(t) \int \frac{dt'}{a(t')}$$

Using the time dependence of  $a$  given earlier this is just

$$d_H/a(t) \sim t^{\frac{1+3w}{3+3w}}$$

This is the horizon distance – the maximum distance that can separate *causally connected* events in the Universe. Notice that ordinary matter ( $w > 0$ ) this distance becomes much less than the size of the Universe  $a$  as  $t \rightarrow 0$ . Thus different parts of the Universe become causally disconnected at early times ...

# Horizon continued

- Thus one would predict that if one looks at widely separated regions of the Universe they should show no correlation in properties if the Universe started in some general state. But this is not what is observed – regions that were never in causal contact appear to be almost identical !! This is the horizon problem.

# Inflation

- Solution – inflation. We assume that early on in the Universe there was a period of very rapid (accelerating) expansion. This requires  $w < -1/3$  as we saw before. But in this case the solution  $\Omega = 1$  is automatically stable and the Universe is naturally flat. Also, this will lead to  $d_H/a \rightarrow \infty$  as  $t \rightarrow 0$  and the horizon is always larger than the scale factor ! Both problems are cured!
- Furthermore, quantum mechanical fluctuations in the early Universe get blown up to large sizes and offer a way of generating the density perturbations one needs to allow large scale structure formation (galaxies)
- These are robust predictions that don't depend on details of high energy physics or issues of quantum gravity.