PHY312 - lecture 15

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Summary of lecture14

- Exterior spacetime to spherically symmetric, time independent, non-spinning gravitational source is given by Schwarzschild metric

$$\Delta s^2 = A(r)c^2\Delta t^2 - \frac{1}{A(r)}\Delta r^2 - r^2\Delta\theta^2$$

  with $A(r) = 1 - \frac{r_S}{r}$
- $r_S = 2GM/c^2$ Schwarzschild radius.
- Global coordinate system. r and t “far away” coordinate and time.
- Visualize at fixed t by embedding in 3D space -> surface. Profile is just $A(r)$.

Consequences

- Controls gravitational redshift, stretching of space.
- If $r_{\mathrm{radius}} < r_S$ spacetime possesses an event horizon

Event horizon

- $A = 0$ at event horizon. Time passes infinitely slowly relative to far away observer.
- Photons emitted from event horizon are redshifted to infinite wavelength and fail to escape to infinity.

$$\frac{\Delta r}{\Delta t} = cA \to 0 \text{ as } r \to r_S$$

- For $r < r_S$ all matter including light must fall to the singularity $r = 0$. Time and space coordinates flip.
- Thus event horizon divides spacetime into 2 causally disconnected regions – stuff inside the event horizon can never influence what happens outside.

What happens at $r = r_S$

- Notice that an observer close to r_S will not notice anything odd – velocity of a radially moving light beam will be c just as per normal:

$$\frac{\Delta r_{shell}}{\Delta t_{shell}} = c \text{ for light}$$

  In fact the metric in shell coordinates is flat ! SR applies.

- He/she will see no change as the light ray crosses the event horizon, no violent redshifting etc
- For a freely falling observer gravity does not even exist - so that there cannot be anything that happens at $r = r_S$ provided that tidal effects are small

Different frames

- Thus while the global coordinates (r, t) tells us (correctly) that something irreversible has happened on crossing r_S it will not seem so to an observer in either a FFF or a shell frame close to the horizon.

FOR in GR

- Only one physical motion through spacetime.
- Many FOR of reference can be used to view it eg shell, FFF, far away (global) frame.
- They agree only on invariant intervals – not things like distance, times, velocities etc.
- Different systems may be better/worse for figuring out different things eg presence of event horizon.
- Notice that shell and FFF are physical frames where observers can make local measurements.
- Global frame different. No real global observer - but does give useful picture of entire trajectory of test particle in spacetime.

Radial stretching done right

- Said that strictly should treat Δr, Δt etc as infinitessimal. But so far taken them finite.
- OK provided Δr small compared to the characteristic scale of the curvature r_S.
- Calculation of Sun stretching fine. But the solar mass black hole ?
- Should really use an integral

$$ds = \frac{1}{\sqrt{A(r)}} dr$$

Hence

$$s = \int_{r_1}^{r_2} \frac{dr}{(1 - r_S/r)^{1/2}}$$

Can be done exactly (or numerically).