PHY312 - lecture 10

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Recap

- FFF serve as good approxs to inertial frames in presence of gravity – Principle of equivalence POE.
- Using POE – learn that light is bent, clocks slow and light is redshifted in gravitational field. Characterized by dimensionless number $\frac{GM}{c^2R}$
- Principle of general relativity – all observers in any FOR should be able to discover the correct laws of physics.
- “Real” gravity is tidal gravity.

More on tidal gravity

- Detect real gravity by watching tidal motion of test particles over finite region of spacetime from FFF.
- Motion does not depend on mass, composition, electric charge etc etc
- Seen that measurements of time suffer distortion in accelerating frames (clock slowing). Similarly one can argue that length measurements are also deformed by gravity/acceleration.
- Worldlines of unaccelerated particles are now bent.
- Is there someway to think of tidal motion as resulting from free motion in a deformed or curved spacetime. What do I mean by this ?

Deforming space by acceleration

- Consider disk rotating at high speed relative to inertial frame. An observer at rest would measure is diameter to be d and circumference C using a ruler at rest in inertial frame. Finds $C/d = \pi$ as usual for flat Euclidean space.
- Observer on disk would measure a distance less than C since he would see ruler as contracted (by SR) along its length. Thus for him $C/d < \pi$! How can this be ?
- Imagine drawing a circle on the surface of a sphere (a curved space). You will find that $C/d < \pi$ in such a case.

Deforming space - II

- But disk is accelerating.
- Thus, if one uses accelerating FOR one can expect to see effects which can be interpreted as resulting from an underlying curved geometry!
- But by POE – same effects should be expected for a disk at rest but sitting in a gravitational field!
- Thus gravity can be pictured as associated with curved space(time)!

Gravity as curved space(time)

- In accelerating frames clocks slow, spatial geometry can become curved.
- By POE, expect that effects of gravity can be similarly associated to an underlying curved curved spacetime.
- Perhaps possible to think of (tidal) motions of test particles in a gravitational field as really free motion in a background curved spacetime.
- Einsteins' guess. It is correct.

Motion on a sphere

- Consider two 2d observers confined to surface of sphere. Set them off a small distance apart at the equator. Let them move freely along lines of longitude.
- You will see that they start to converge and eventually meet at the North Pole – just like for tidal gravitational motions....!
- Indeed, if they didn't know they lived on a curved space they might infer that a myesterious force (gravity) was drawing them together.
- Reality is that they are moving along "straight lines" in an underlying curved space.

What causes curving ?

- Take a hint from Newton. Mass causes Newtonian gravity. Presumably some relavistic generaliation of mass is responsible for spacetime curvature - energy-momentum ....
- Nice picture. Force of gravity is just curvature of spacetime. But clearly this curving is independent of the coordinates one uses on the spacetime (I can use a variety of different coordinates systems to describe the surface of the sphere).
- Thus physical laws should not depend on the coordinate system used to describe the underlying space – or equivalently – all coordinate systems (frames of reference) should be equally good choices for describing the curvature of spacetime. This is just the principle of general relativity again!