

## PHY312 - Homework 9

1. A black hole has an event horizon size of  $10\text{km}$ . A stone starting from rest at infinity falls radially into the black hole.
  - (a) At what speed does a shell observer at  $r = 35\text{km}$  measure the stone to be going as it passes him ?
  - (b) What is the Schwarzschild (i.e global frame) speed of the stone as it passes  $r = 35\text{km}$ ?
  - (c) What are the values of the shell speed and Schwarzschild speed at  $r = 15\text{km}$  ?
  - (d) What are these speeds at the event horizon ?
2. A spaceship uses its rocket motors to remain stationary at r-coordinate  $r = 4 \times 10^6\text{km}$  around a supermassive black hole at the center of a galaxy. The Schwarzschild radius (event horizon) of the black hole corresponds to  $r = 1 \times 10^6\text{km}$ .
  - (a) Suppose the ship's engineer drops a wrench. What is the magnitude of the observed acceleration of the wrench toward the floor ? (Hint: use the expression in the lecture notes for  $\frac{d^2 r_{shell}}{dt^2_{shell}}$ ) Do you think the engineer is human ?
  - (b) Estimate the tidal acceleration on an object of size 1m at this position (Newtonian expressions will do).
3. Imagine releasing an object from a stationary position at r-coordinate  $r = 20\text{km}$  above a five solar mass black hole.
  - (a) What is  $\frac{dr}{dt}$  - the global Schwarzschild speed as the object approaches a distance of 1 km above the event horizon of this black hole.
  - (b) Using the relation between  $r_{shell}$  and  $r$  and  $t_{shell}$  and  $t$  calculate the speed the object would be moving at this point to a local (shell) observer.
  - (c) What shell speed does the object attain at the event horizon. Does this depend on  $r_o$  ? Comment.
  - (d) What is the Schwarzschild velocity at 1km above the event horizon for a proton traveling towards the hole from infinity if its initial speed is 99.99% speed of light.