PHY312 - Homework 7

1. Consider a solar mass black hole ($M = 2 \times 10^{30}$kg). An observer standing on a shell with r coordinate $r = 5$km shines his flashlight radially outward. The light produced by the flashlight has wavelength 4×10^{-7} m (violet light). What is the wavelength of the light seen at infinity ? What is its approximate color ?

2. A black hole is thought to be empty except for a point of infinite density - the singularity. All the matter that makes up the mass of the black hole ends up there. Nevertheless, one can define an average density for the black hole by taking the total mass and dividing by the volume of the sphere whose radius is the Schwarzschild radius.

   (a) Write an expression for the mass of this black hole in terms of the average density and r_S.

   (b) Write another expression for the mass of the black hole in terms of G the gravitational constant and c the speed of light and r_S.

   (c) Equate these two expressions and hence write down an expression for the Schwarzschild radius in terms of the average density and G and c.

   (d) Suppose one creates a black hole whose average density is about the same as the Earth's atmosphere - 1 kg/m^3. How big is the Schwarschild radius ? Compare your answer to the radius of Pluto's orbit 6×10^{12} m. Supermassive black holes with large r_S are thought to inhabit the core of many galaxies including our own. (Take $G = 6.7 \times 10^{-11}$ and $c = 3 \times 10^8$)

   (e) How many solar masses is the black hole we have discussed here ?

3. Tidal gravitational effects in Newton's theory are given by an expression $F_{\text{tidal}} = 2GM/r^3\Delta r$. It is the magnitude of these effects that cause problems for an observer freely falling into a black hole. What is the magnitude of these tidal effects as one crosses the event horizon of

   (a) A five solar mass black hole

   (b) The supermassive black hole constructed in the previous question.

   Assume that $\Delta r = 1$ the scale of the observer's body.