PHY312 - Homework 10

1. Consider a satellite in a circular orbit around a black hole at r-coordinate r_o.

   (a) What is the angular momentum per unit mass in terms of v_{shell} ? (Hint: the answer is **not** $r_o v_{shell}$)

   (b) The satellite does one orbit and returns to its initial position. This distance is $2\pi r_o$. How much time does this take on a stationary shell clock (in terms of v_{shell}) ?

   (c) What is the time taken as measured by a clock on the satellite ? (Hint: this is the proper time elapsed i.e this question is asking about time dilation)

   (d) How much time would the orbit take as recorded by a "far-away" observer? (Hint: use the relationship between dt and dt_{shell})

2. From the formula given in class for the radius of stable circular orbits show that the smallest stable circular orbit occurs for $r = 3r_S$ where r_S is the Schwarzschild radius. There are no stable orbits closer to the black hole than this.

3. A satellite is fired at 90^0 to the radial direction at r-coordinate $r_0 = \frac{8GM}{c^2}$ at a local (shell) speed of $v_0 = \frac{3c}{4}$. What is the angular momentum per unit mass and energy per unit mass of the satellite ? Will this satellite escape to great distance ? (Hint: plot a picture of the effective potential for this satellite and draw on this picture the satellite's energy)

4. Write down an expression for the effective potential for photons in a Schwarzschild geometry. Find the radius of *unstable* circular orbits in terms of GM/c^2 (the radius of the *photon sphere*). If the impact parameter $b = \frac{6GM}{c^2}$ will the photon be captured by the black hole ?