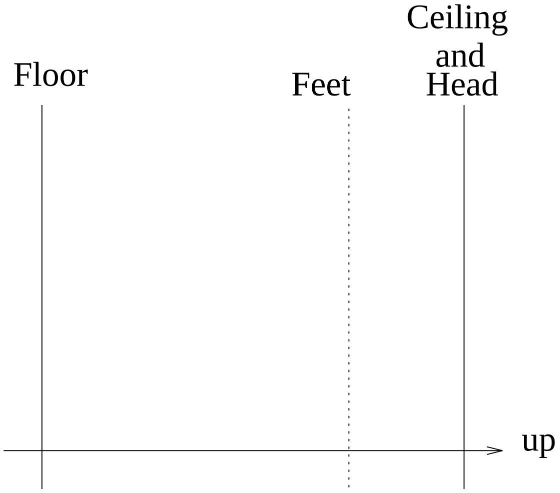
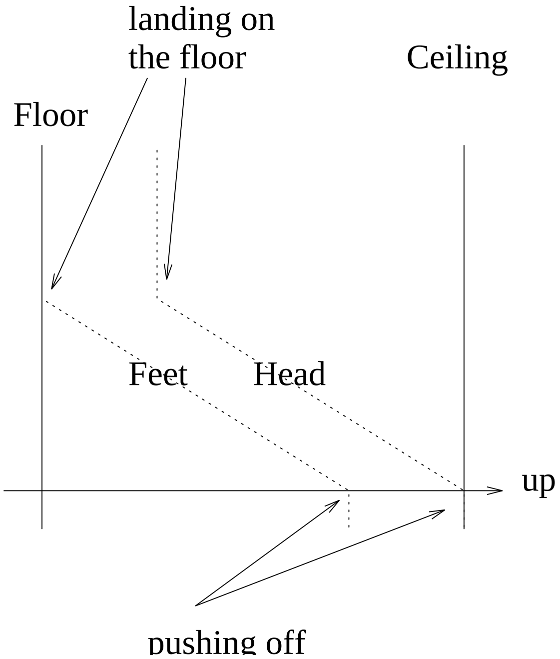
Solutions to homework Assignment #2
PHY312 – Spring, 2003

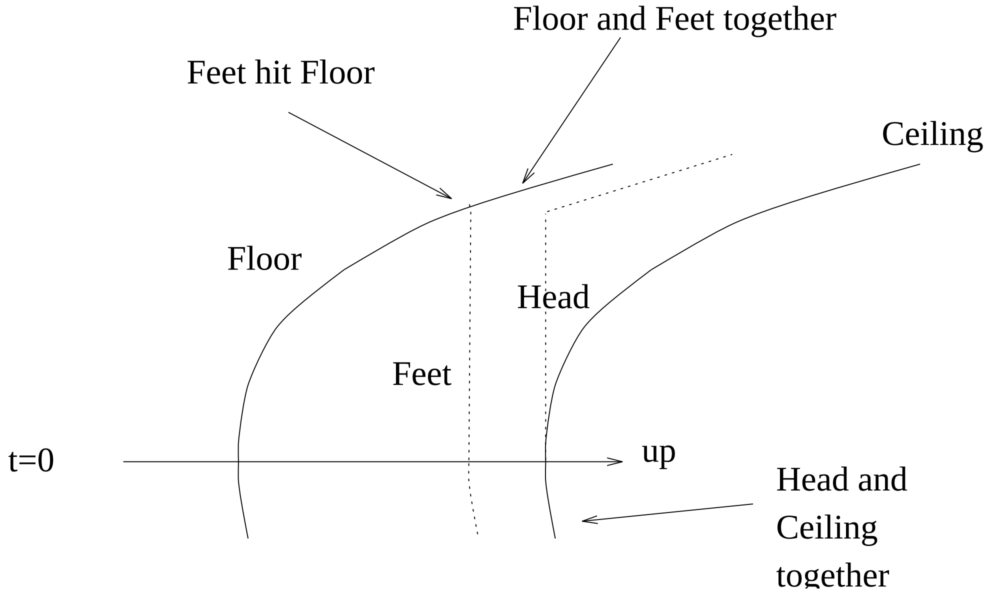
1-8. a) In this case, the rocket engines are off. So, *the rocket is in an inertial frame.* Before you let go, your reference frame is that of the rocket and is also inertial (so that no force acts on you). Furthermore, since you simply let go, the total force on you *always remains zero.* Thus, your frame of reference is inertial too. Note that your relative velocity with respect to the rocket is zero at $t = 0$ (since you were holding on to it before then). Since both you and the rocket remain in inertial frames, the relative velocity will be constant. That is, it will remain zero at all times. So, the inertial frame that is at rest relative to you is just your own frame of reference, and it is also the rocket's frame of reference. Nothing moves in the corresponding diagram. This means that it's pretty easy to draw – we just have to make sure that we make 'up' the proper direction (that is, the ceiling is more 'up' than the floor, and your head is more 'up' than your feet.) Note that, for this case, your head and the ceiling always remain together.



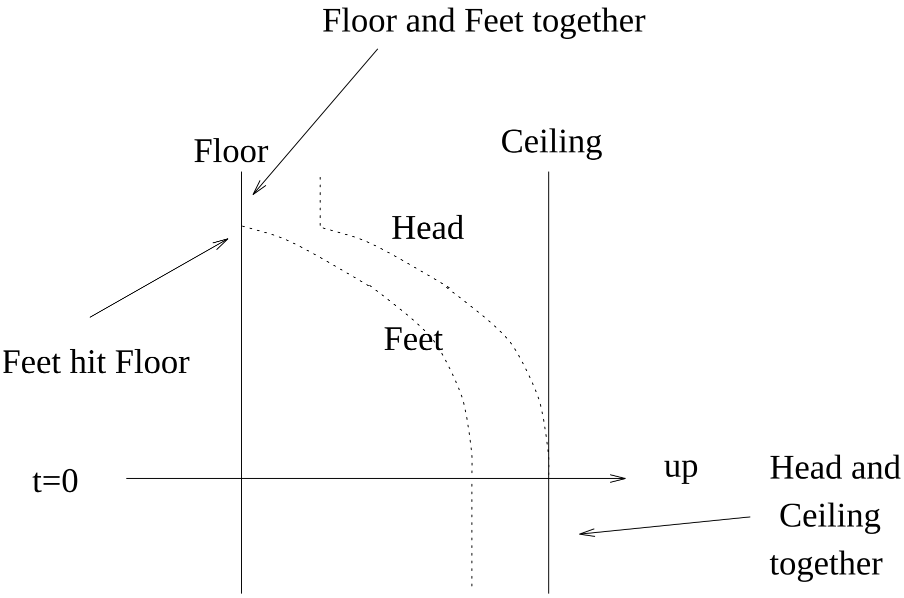
b) This time, you give yourself a push. During the push, a force acts on you an you are not in an inertial frame. However, the rocket still is. [Note: technically speaking, it's not – because you're pushing on it!!! However, rockets are big and heavy. Even though you push on it, it will still remain (almost) in an inertial frame.] So, the *inertial* frame of reference that is at rest relative to you just *before* you push off is the frame of the rocket. Since you push off, your velocity will no longer be constant in this frame – it will change during the time your are pushing away. Once you stop pushing, of course, there will again be zero force acting on you and you will be an inertial object moving at a constant velocity in the rocket's frame. Of course, when you hit the floor a force will again act on you, and your velocity will change again. Assuming that you hit the floor gently and don't fall down (so that your head stays a full body length above your feet), the picture looks like this:



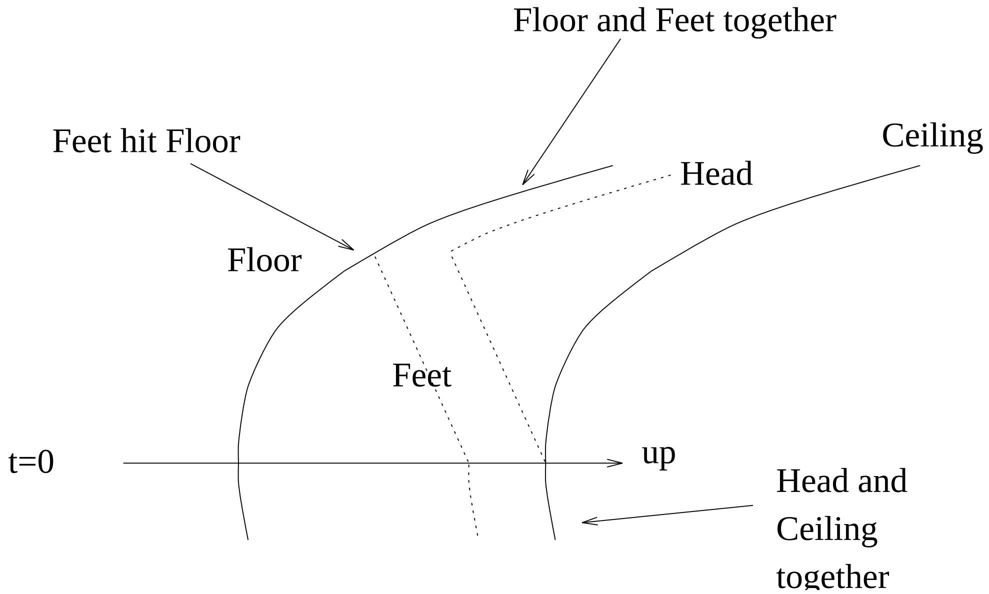
c) This one is much like (a), except that the rocket engines are *on*. This means that the rocket will *not* have an inertial frame of reference. It also means that *while you are holding on to the rocket or standing on the floor* your frame of reference will not be inertial either. However, while you are "falling" nothing will be touching you and so no forces can be acting on you. During this time, you will be in an inertial frame, and you will follow a straight worldline on the diagram. In fact, recall that you were to draw the diagram in an "inertial frame of reference that is at rest relative to you just before you let go." Let's call this time $t = 0$. Since no forces act on you after $t = 0$, your velocity relative to this frame will not change and you will remain at rest relative to this frame. [Note, however, that you are *not* at rest with respect to this frame before you let go or after you land. So, the frame of reference in which the diagram is to be drawn is *not* the frame of reference of any actual object mentioned in the statement of the problem. This is perfectly OK: we can either think of some abstract inertial frame which is not the reference frame of any particular object or, to be more concrete, we could just imagine some object (like a rock or a tin can) which just happens to be floating outside the rocket and use its frame of reference.] Thus, we will draw both your head and feet as straight vertical lines during the time you are "falling." The rocket, however, is accelerating 'upward.' So, the worldlines of the ceiling and floor will curve in the 'up' direction:



d) This is the same physical situation as (c), but now we are to draw the diagram in the reference frame *of the rocket*. As noted above, this is *not* an inertial frame. However, that is OK. At least in Newtonian physics, we know how to draw diagrams in accelerated (non-inertial) frames of reference. The first key point here is that, since we will work in the rocket's frame, the rocket's worldlines will be straight and vertical. The other key point is that, at each time, the separation between, say, the ceiling and your head, can be read directly off of the diagram above. So, we can use this observation to translate the worldlines of your head and feet onto our new diagram:



e) This case is the same as (c) except that you give yourself an extra push away from the ceiling. So, this will be a sort of combination of (c) and (b), where the extra push puts an extra bend in your worldlines. The result looks like:

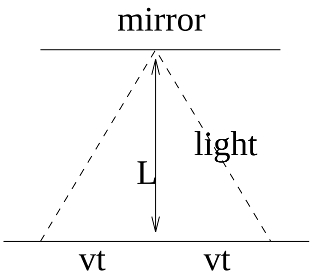


Note that, this time, you hit the floor a little sooner.

f) Only in case (a) are you *always* in an inertial frame. In (b) and (e) there is a nonzero force acting on you while you push yourself away from the ceiling and, in (c) and (d) there is a nonzero force acting on you while you are holding on to the ceiling (before you fall) and while you are standing on the floor (after you fall).

2-1. $\epsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{Nm^2}$ and $\mu_0 = 4\pi \times 10^{-7} \frac{Ns^2}{C^2}$. So, $\sqrt{\epsilon_0 \mu_0} = .3336 \times 10^{-8} \frac{s}{m}$ and $1/\sqrt{\epsilon_0 \mu_0} = 2.998 \times 10^8 m/s$.

2-2. For this problem, let's look at what happens in the ether's frame of reference. First, let's think about the bit of light that goes out and back along the vertical arm. Recall that, from the ether's point of view, the earth (and thus the whole apparatus) will be sliding to the right during the time that the bit of light is moving. So, when the light reaches the mirror at the top, the whole thing (including the bit of light) must have moved some distance to the right. This means that, in the frame of reference of the ether, the light does not just move up and down. The path followed by the light (in the ether's frame) looks something like this:



Here, I have let t be the time it takes the bit of light to move up and hit the top mirror (which is, by symmetry, also the amount of time it takes the light to come back down). As I have marked on the diagram, during the trip up, the device (and thus the light) has slid over a distance vt to the right. It slides over the same distance again during the light's trip back down. Note that the total distance covered by the light is given by the Pythagorean theorem: on the way up, it is $\sqrt{L^2 + (vt)^2}$. But, it travels at speed c for a total time t, so we must have

$$c^2t^2 = L^2 + v^2t^2.$$

Solving this equation for t yields $t = \frac{L}{c\sqrt{1-(v/c)^2}}$. So, the total time it takes the light to go up and back is $T_1 = \frac{2L}{c\sqrt{1-(v/c)^2}}$. We could use the last approximation given in the statement of the problem (with $x = v/c$ as suggested) to write this approximately as

$$T_1 = \frac{2L}{c(1-\frac{1}{2}(v/c)^2)} = \frac{2L}{c}(1+\frac{1}{2}(v/c)^2).$$

Now we need to think about what happens along the horizontal arm. Notice that, as the light moves to the right, the mirror tries to run away from it, since the whole device is moving to the right. Of course, the light is faster and eventually catches up. Even so, the light had to move farther than just a distance L. If it took a time t_1, then the light had to move a total distance of $L+vt_1$. Since the light moves at speed c, we have $ct_1 = L+vt_1$ or $t_1 = \frac{L}{c}\frac{1}{(1-v/c)}$. The trip back is much the same, except that now the left end is running *toward* the oncoming light, so the light travels a distance less than L. If it takes a time t_2 for this trip, we have $ct_2 = L - vt_2$ or $t_2 = \frac{L}{c}\frac{1}{(1+v/c)}$. So, the total time to go out and back along the horizontal arm is

$$T_2 = t_1 + t_2 = \frac{L}{c}\left(\frac{1}{1-v/c} + \frac{1}{1+v/c}\right).$$

Using the approximations, we get

$$T_2 = \frac{L}{c}\left(1 + v/c + (v/c)^2 + 1 - v/c + (v/c)^2\right) = \frac{2L}{c}(1 + (v/c)^2).$$

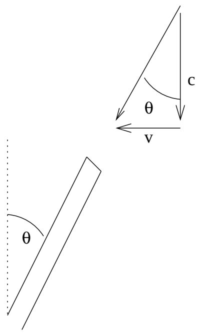
Subtracting these two, we have

$$T_2 - T_1 = \frac{L}{c}(v/c)^2.$$

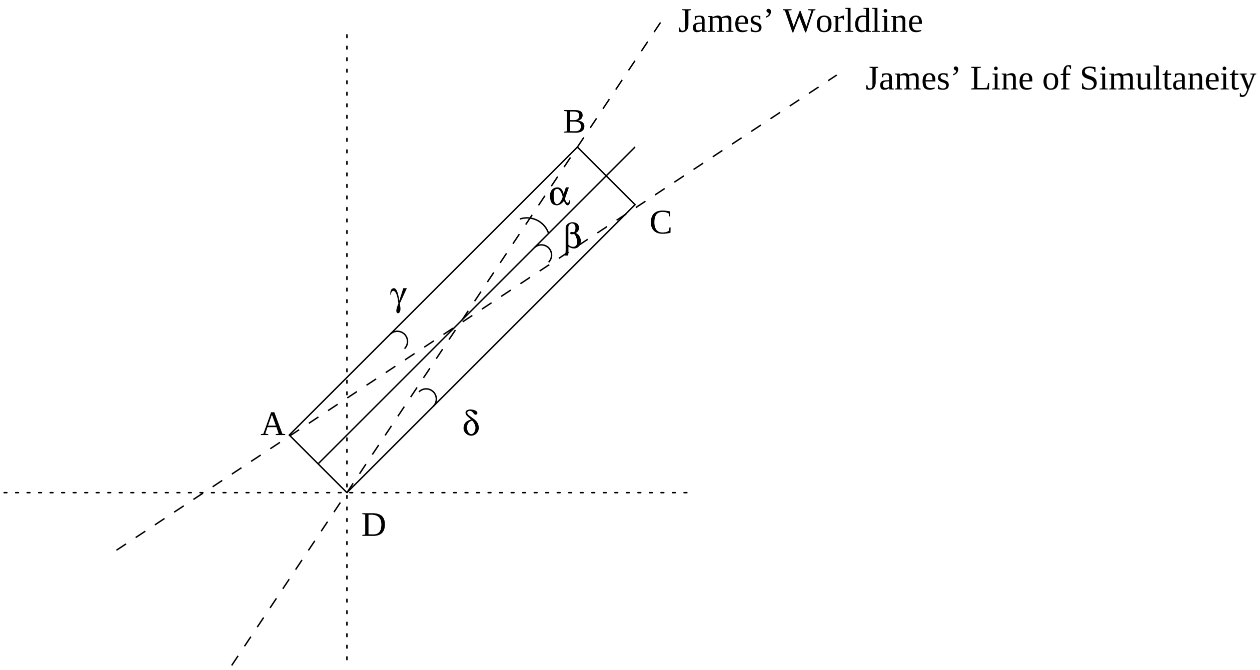
The time that light takes to travel the horizontal arm is indeed larger, by the amount $\frac{L}{c}(v/c)^2$.

2-3. In the reference frame of the telescope, the light is moving down at speed c and *to the right* at speed v. Thus, we must point our telescope at an angle θ for which $\tan\theta = v/c$ as shown in the diagram below. Thus, $\theta = \tan^{-1}(v/c)$.

In the telescope's frame:



3-1. Let me begin by redrawing the spacetime diagram for this problem. The dotted line below denotes one reference frame and the dashed lines denote another reference frame (let us say, for example, the reference frame of James [J]). The solid lines are light rays (which, by our convention, always travel at 45 degrees with respect to the vertical. Recall that James' line of simultaneity and James' worldline both pass through opposite corners of the rectangle (ABCD) made out of the light rays.



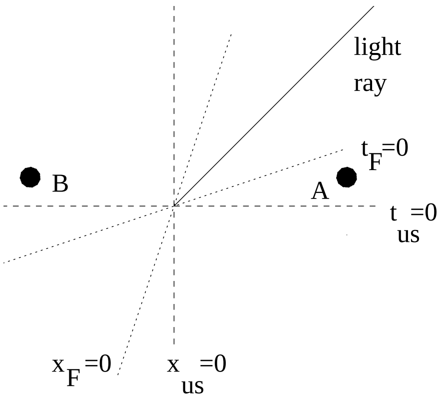
Probably the easiest way to proceed is to notice that, since all of the light rays that slope up and to the left are parallel, the following angles are equal: $\alpha = \delta$ and $\beta = \gamma$. Thus, all we need to do is to show that $\gamma = \delta$.

However, the two triangles (ABC) and (BCD) are both right triangles since any two light rays intersect at a 90 degree angle. (That is, the angles ABC and BCD are both 90 degree angles.) Furthermore, since (ABCD) is a rectangle, the two sides (AB) and (CD) are of the same length. The length of (BC) is clearly the same as itself, so it follows that, in the triangle (ABC) and (BCD) we can match up two sides and a corresponding angle. This means that the two triangles are congruent, so that the angles γ and δ are equal. QED (Note: Instead of talking about congruent triangles, this argument can also be done using trigonometry:

$$\tan\gamma = \frac{BC}{AB} = \frac{BC}{BD} = \tan\delta, \tag{1}$$

so $\beta = \gamma = \delta = \alpha$.)

3-2. This problem is much like what we were talking about in class on Tuesday. Let's start by just drawing our friend and our friends' line of simultaneity (dotted lines) on our spacetime diagram. We can, for example, determine our friend's line of simultaneity from the rule you derived in problem 3-1.



Our worldline and our line of simultaneity ($t_{us} = 0$) are the dashed lines, and I have drawn in a light ray (solid line) for reference purposes. I have labeled two events, A and B above. What I did was just to draw our friends' line of simultaneity and pick two events that are simultaneous as defined by *us* – that is to say which lie on the same horizontal line. I then noted that one of the events is *before* (below) our friends' line of simultaneity ($t_F = 0$) and I labeled that one A. The other is after (above) my friend's line of simultaneity, so I labeled that B. Clearly, to get B before A according to my friend, I just have to reverse the relative velocity: