Solutions to homework Assignment #1
PHY312 – Spring, 2001

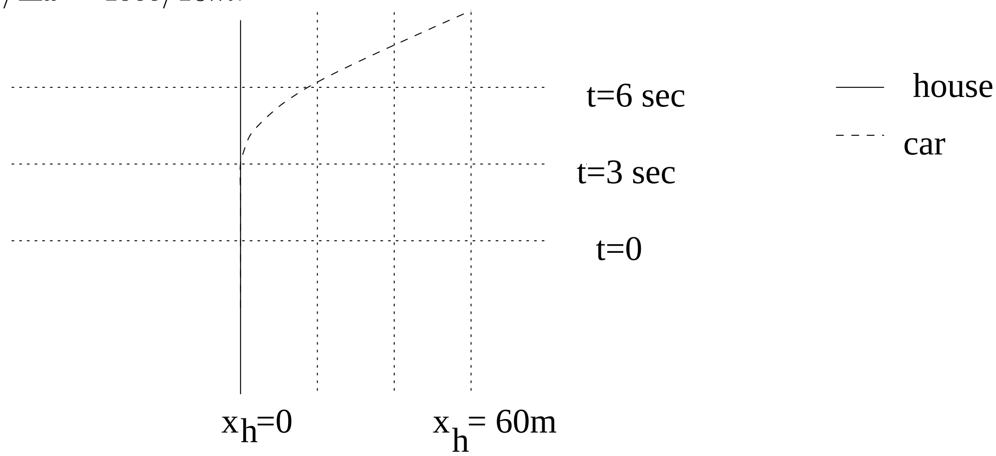
0. *Solution to logic problem from the survey:* What your friend said was that *each* card that has an A on one side has a 4 on the other. So, if we wish to check her claim, we can break our task into two parts. We must first find all of the cards with an A on one side, and then check that they have a 4 on the back. Cards that do not have an A on them are irrelevant and can be ignored.

- Card 1 shows A: Clearly, we must check the back of this card to see if it has a 4.
- Card 2 shows 7: We do not yet know if this card has an A. If it does, then our friend is wrong. If it does not, then our friend may still be correct, provided no other problems arise. Thus, we must check the back of this card to see if there is an A or not.
- Card 3 shows E: Since we see the E, this card has no A. This means that we can ignore it.
- Card 4 shows 4: If this card has an A on the back, then it does not disprove our friend (since it has a 4 as well). If the card does not have an A on the back, then we have already agreed that it is irrelevant. Thus, in both cases it does not disprove our friend and we may ignore the card.

As a result, we need to flip the A and the 7, but not the E or the 4.

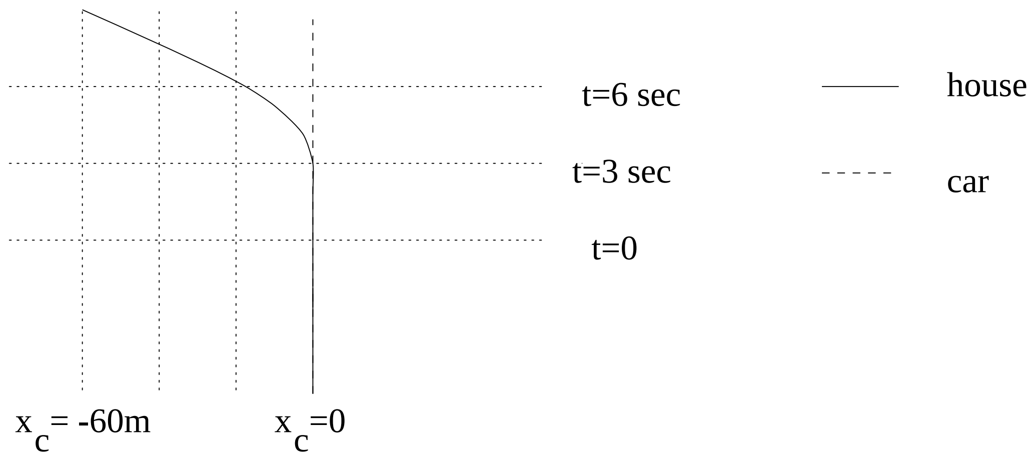
1-1. OK, let's start with the house's frame of reference, since that's how we're used to thinking of this process. Of course, drawing the worldline of the house is easy – since we work in the house's frame of reference, it stays right at $x_h = 0$ (The subscript h is for house). The worldline of the house is the solid line on the diagram below. Now, the house finds the car to be initially stopped ('at rest') and parked right outside. So, at the beginning, the car also stays right at $x_h = 0$. Let's say that you start the car at $t = 0$. If you wait a few seconds before stepping on the gas (at say $t = 3$ seconds), the car will stay at $x_h = 0$ until $t = 3$ seconds. We'll draw the worldline of the car as the dashed line below.

At $t = 3$ seconds, of course, the car starts to pull away. At first, it is moving very slowly, so it's worldline remains nearly vertical (it covers very little distance as time passes). But then it goes faster and faster, so the worldline tilts over more and more. Eventually, the car is going at a constant $30mph$ (=$13m/s$), so its worldline straightens and attains a slope $\Delta t/\Delta x = 1sec/13m$.

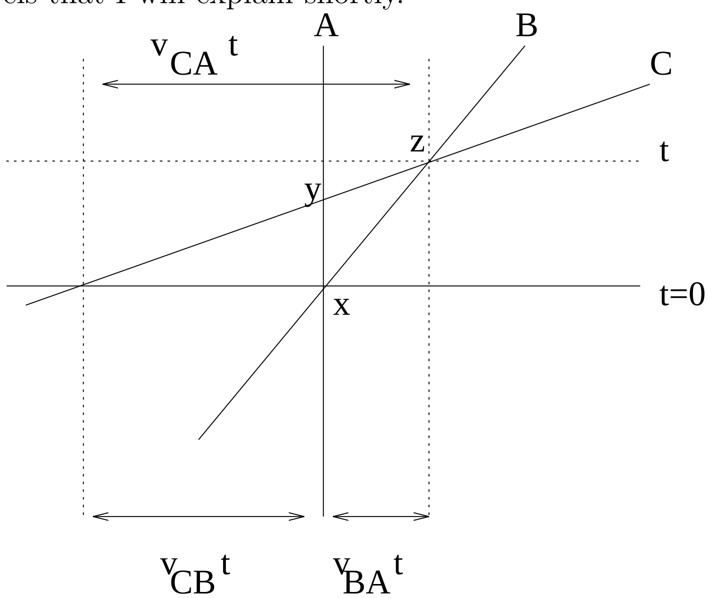


Our next task is to draw this whole scenario in the reference frame of the driver of the car. Again, we'll use a solid line for the house, and a dashed line for the car. Of course, in the driver's reference frame, both the driver and the car itself will remain at $x = 0$ for all time. So, the only part we have to think carefully about is the worldline of the house. It is perhaps easiest to work this out using the diagram we already drew above. At each time t, we can read off the separation between the car and the house. For example, until $t = 3sec$ the car and the house are at the same location. Since the car is at $x = 0$ on our new diagram, we also draw the house at $x = 0$ until $t = 3sec$.

Now, after $t = 3sec$, our diagram above shows that the house and car begin to separate, with the house being to the left of the car. We can read off the separation at each time, and use this to graph the worldline of the house in the driver's frame of reference. For example, at $t = 6seconds$, we see from the above that the car is almost $20m$ to the right of the house. Thus, at that time, in the reference frame of the car, the house is very close to $x = -20m$. The minus sign is because the house is to the *left* of $x_c = 0$ (the car). In the same way, we can read off the separation at any time t from the diagram above and copy it (reversing right and left) to the new diagram. The result has been drawn below.



1-2. We can work this out just the same way that we did the simpler case (where all three worldlines did pass through the same event) in class. The spacetime diagram from the problem is shown below. I've added some extra labels that I will explain shortly.



For convenience, we have chosen the time $t = 0$ to be when A and B meet, and I have labeled their meeting 'event x.'. I have labeled the meeting of A and C 'event y' (and will call the corresponding time t_y) and the meeting of B and C 'event z' (with the corresponding time, t_z).

We will proceed much as we did in class. Using property T, we can freely refer to 'the time x, y, or z' without stating who measures this time or how the measurement is performed (let's assume that all clocks have been synchronized). Similarly, we can use property S to talk about the distance between two events at the same time without worrying about who performs the measurement or how it is done. That is to say, using properties T and S we can talk about, say, how far the worldline of A is from event z at time t_z, and this will be the same whether it is measured by A, B, or C. Since A finds B to move by at velocity v_{BA} (and since A and B were together at time $t = 0$), this is just $v_{BA}t_z$. This distance has been labeled on the diagram above.

Let us now ask how far apart B and C are at time $t = 0$. Note that, during the time interval between t=0 and time $t = t_z$, object C will just catch up with object B. Thus, the distance separating B and C at time $t = 0$ must be just $v_{CB}t_z$. (Note that $v_{CB} > 0$ while $v_{BC} < 0$.) But, of course, B and A are at the same place at $t = 0$, so this is also the distance between A and C at $t = 0$. This distance has also been marked on the diagram.

On the other hand, let us consider the total distance that object C moves (as seen by A) between 0 and t_z. This must be $v_{CA}t_z$. From the diagram, it is clear that we have:

$$v_{CA}t_z = v_{CB}t_z + v_{BA}t_z,$$

since the two double arrows representing the distances $v_{CB}t_z$ and $v_{BA}t_z$ can be put together to make $v_{CA}t_z$. Canceling the t_z's, we arrive at $v_{CA} = v_{CB} + v_{BA}$ as desired.

1-3. As we discussed in class, an object is in an inertial reference frame *if and only if* that object has zero total force acting on it. This point will help us to answer this question.

a) Since a rock in (very) deep space is very far away from everything else, there is nothing around to exert a force on the rock. Thus, the total force on the rock must be zero!! It follows that the rock is in an inertial frame of reference.

b) Now, when a rocket has its engine on, the engine provides a force on the rocket (technically, this is a force of the exhaust pushing on the rocket). Since the rocket is off in deep space, nothing else can exert a force on the rocket – so nothing can cancel out the force from the engine. As a result, the total force on the rocket is definitely *not* zero, and the rocket is *not* in an inertial frame.

c) We could answer this one in either of two (closely related!!) ways. One way is to note that the earth exerts a gravitational force on the moon, so that the total force on the moon is not zero and the moon cannot be in an inertial frame. Alternatively, we could recall that the earth is (more or less) in an inertial frame and that, viewed from the earth, the moon goes around and around in circles. In particular (as viewed from the earth), the moon does *not* travel in a straight line with constant velocity!!!! We again see that the moon is not in an inertial frame.

1-4. This problem is easiest to attack by recalling that, given an inertial frame E (say, that of the surface of the earth [which is 'inertial enough' for this problem]), the other inertial frames are exactly those that move in a straight line at constant velocity relative to E.

a) A person standing on the ground is not moving with respect to frame E. Zero velocity *is* a constant speed (zero) along a straight line. So, the person is in an inertial frame. (At least, one that is 'as inertial' as that of the earth.)

b) Since the person goes both up and down, they do not always move in the same direction (with respect to frame E). First they move up, then they move down. Thus, this person is *not* in an inertial frame.

c) Since the car is going around a curve, it is not moving in a straight line (relative) to E. Thus, this person is also not in an inertial frame.

d) Since this car and person are moving at a constant speed in a straight line (with respect to E), this reference frame is inertial.

1-5. Let's first think about what this problem is asking. Note that it says 'suppose that Newton's second law is true in (inertial) frame A.' It then goes on to explain further what the statement in quotes means. It means that you should begin by assuming that the equation $F = ma_{CA}$ is true. Note that this refers to the acceleration of C as measured by A. What the problem wants you to do is to start from this equation and, using properties of inertial frames that you know (from class), show that the equation $F = ma_{CB}$ (which refers to the acceleration of C as measured by B this time) is also true.

Well, perhaps the first thing to remember is that the force F and the mass m do not depend on the reference frame. So, what we would really like to show is that $a_{CA} = a_{CB}$. If we knew that this were true, we would very quickly arrive at the result we want. The question is, how can we arrive at this relation between the acceleration of C and measured by B and the acceleration of C as measured by A?

Let's start with the question "what do we know about how A's and B's measurements of C are related?" We have talked a lot (see problem 1-2) about how their measurements of C's velocity are related:

$$v_{CA} = v_{CB} + v_{BA},$$

where v_{BA} is the velocity of frame B relative to frame A. Of course, we don't know exactly what v_{BA} is but, since both A and B are inertial frames, we *do* know one important thing about v_{BA}: the relative velocity of any two inertial frames is *constant*.

Now, acceleration is the time derivative of velocity. So, if we take the time derivative of both sides of the above equation, we can find out how the accelerations are related. That is, we use $a_{CA} = \frac{d}{dt} v_{CA}$ (etc.) to find:

$$a_{CA} = a_{CB} + a_{BA}.$$

But v_{BA} was *constant* so $a_{BA} = \frac{d}{dt} v_{BA} = 0$. Thus, $a_{CA} = a_{BA}$ and, if $F = ma_{CA}$, then we also have $F = ma_{CB}$. QED

1-6. There are many, many, points that could be made here. For example, we saw in problem 5 that the consistency of the first and second laws depends on the formula $v_{CA} = v_{CB} + v_{BA}$ for the addition of velocities, which was true only because of T and S. Or, we might note that Newton's law of universal gravitation simply refers to the distance between two objects, without saying anything more about how this distance is measured. This is a clear use of assumption S (that any two ways of measuring the distance agree). Similarly, we could notice that this law gives the force *at some time* and that it depends on the distance *at that same time*. As we have discussed, the notion of 'the same time' is well-defined only due to assumption T. This last point is perhaps the most common implicit use of T is Newton's laws ... for example, note that Newton's second law really says that the force on an object *at some time* is proportional to the acceleration of the object *at that same time*.

1-7. OK, let's start by recalling the precise statement (that I gave) of Newton's first law: There exists a class of reference frames (called inertial frames) in which an object moves in a straight line at constant speed *if and only if* the total force on that object is zero.

a) First let's prove that (1) implies (2). So, let's suppose that we do in fact have an object which is in an inertial frame of reference. We may note that, in it's own reference frame, it always remains at the same location (usually taken to be $x = 0$). Thus, in this reference frame, it moves at zero speed. Zero is a constant speed along any straight line so, since this reference frame is inertial, Newton's first law says that the object must be experiencing zero total force. Thus, by assuming (1), we have derived (2). That is, (1) implies (2).

b) Now let's show that (2) implies (3). This follows trivially from Newton's first law: The total force on the object is zero, so in any inertial frame it moves at a constant speed in a straight line.

c) Finally, let's show that (3) implies (1). This one is perhaps the most subtle. Let us assume that our object (let's call it A) moves at a constant speed in a straight line (that is, constant velocity) in any inertial frame. Let's consider its motion in a particular inertial frame (call it B). Then what we have is the statement that v_{AB} is constant. However, $v_{BA} = -v_{AB}$, so v_{BA} must be constant too. This observation will be useful later.

Now, according to Newton's first law, in order to see if A's frame is inertial, we have to consider objects that are *known* to have zero total force on them move and see how they move with respect to our object A. So, let's think about this. Consider *any* object with zero total force on it (call this object C). We already know that C will have constant velocity with respect to B. Thus, v_{CB} is constant. Since this argument is in the context of Newtonian mechanics, we can use the relation $v_{CA} = v_{CB} + v_{BA}$ to see that this means that v_{CA} must be constant too. Thus, the object (C) with zero total force on it moves at constant velocity in the reference frame of A. Since this was true for *any* such object (C), Newton's first law says that A has an inertial frame of reference. QED

By the way, note that we explicitly used certain relations among the velocities that are true only if those old assumptions (S and T) about space and time are true. As we now know, they are not. So, how would we proceed in post-Einstein relativistic physics? (This is now beyond what the homework problem was asking for.) There is of course a formula telling how the velocities v_{CA}, v_{BA}, and v_{CB} are related in relativity. However, in order to derive that formula, we will need the result we are trying to prove *here* that any object which moves at constant velocity as measured by an inertial frame is also in an inertial frame. So, we can't use that formula yet.

The trouble is that, logically speaking, it is difficult to show (using our assumptions so far) that there is more than one inertial frame! However, experience and experiments (like those of Michelson and Morely) tell us that

that there are many inertial frames, and that, given any inertial frame B and any velocity v (say, with $v < c$, there is an inertial frame C whose velocity (at this moment) with respect to B is v. So, let us take this as an additional assumption.