

# Relativity and Cosmology: Einstein and Beyond!

Prof. Donald Marolf

April 22, 2003

## 1 Introduction

*There once was a lady named bright  
who traveled much faster than light.  
She left home one day  
in a relative way  
and returned the previous night.*

This physics limerick captures an important fact about Einstein's theory of special relativity: according to Einstein, traveling faster than light does not just allow time travel, it is actually *equivalent* to time travel in a certain sense. You may not understand this now (since the course has not yet started) but you will by the end of the semester. This feature, time dilation, the famous  $E = mc^2$ , curved spacetime, black holes, and cosmology are some of the nifty things we'll be discussing in this course.

However, it is important to stress that this is not a course in science fiction. Relativity is an intrinsic part of such real-world phenomena as the Global Positioning System (GPS), nuclear energy, and the Laser Interferometric Gravitational wave Observatory (LIGO). All of the topics mentioned in the preceding paragraph are important physics concepts and we're going to discuss them *correctly*. Unfortunately, most sci-fi authors get these things wrong, so you'll have to try to forget anything that you may have 'learned' from sci-fi. (Trust me, you'll never watch or read science fiction the same way again after this course!!)

So, what does it take to understand these things properly?? There are two answers: 1) patience and 2) very careful thinking. The topics studied in this course are so far from our everyday experience that the intuition you have built up during your life so far will probably not be helpful to you – in fact, it will be your biggest hindrance. Many of the things we will study are counterintuitive, but this does not mean that they are wrong, unobservable, irrelevant to the 'real world,' or un-understandable. It is important to remind yourself of this early and often and to think very carefully about what you mean when you say (or are tempted to say) that something does or does not 'make sense.' The way

to survive in a realm where your intuition does not apply is to learn to think clearly and accurately – that is, to base your reasoning on careful logic – and to build carefully from the few experimental facts you do have.

## 1.1 A Small Warning

Unfortunately, I feel the need to start with a small warning. My assumption is that students take this course because they are interested in the subject and would like to learn the material. I also assume that students are willing to apply themselves seriously to this task. Although the material is challenging, it can be mastered by anyone who can understand a bit of calculus. While it has sometimes been a struggle, there has never been a case of a student working hard in this course who was unable both to understand the subject and to perform well.

On the other hand, this is by no means a blow-off course. Occasionally, students wander into this course under the mistaken impression that this will be an easy way to fulfill a science elective for, say, an engineering degree. Perhaps it is the fact that this course is billed as being accessible to freshmen or the fact that it is billed as de-emphasizing mathematics (in comparison to most treatments of relativity) that draws such students. If you came with this mind, my advice is to either to get out now or to prepare yourself for a serious experience. While this course is not designed to be a bone-crusher, and while I do believe that it can be an excellent course even for non-technical people and non-scientists, Relativity and Cosmology should not be taken lightly.

I have put real work into this course and I will expect you to do the same.

## 1.2 Course Goals

The Goals for this course are:

- 1) To introduce the basic concepts of special and general relativity and to provide some skill in their use.
- 2) To promote skill in clear, precise, and analytical thinking.
- 3) To provide practice in altering one's opinions and intuitive picture of a structure in light of new evidence. In this case, the structure is none other than the 'well-known' framework of space and time.

## 1.3 Course Objectives

The following is a list of some specific skills that you will gain during the coming semester. At the moment, you may not even know what any of these mean but, by the end of the semester you should be able to:

- 1) Read and interpret a spacetime diagram.

- 2) Draw your own spacetime diagrams and be able to manipulate them to answer questions about and understand a number of effects in relativity.
- 3) Understand and be able to use the (local) equivalence principle; understand the basic properties of curved spacetime.
- 4) Describe the basics of time dilation, length contraction, cosmological models, and black holes.

## 2 Administrative Info

**Lectures:** Tuesdays and Thursdays 11:30-12:50, Physics Building Rm106. Trust me: you will want to bring a set of colored pens or pencils to class in order to work effectively with complicated diagrams.

**Instructor:** Don Marolf (Office: 265-3 Physics, office phone: x3882, home phone: 422-3902 [Note: My home phone number is incorrect in the syllabus in the photocopied Notes from the bookstore. This one is correct!!], e-mail: marolf@physics.syr.edu)

**Office Hours:** Tuesdays 1:00pm - 2:30pm in Physics 263-5 and Wednesdays 7:00pm - 8:00pm+ in Physics 204. On Wednesday, I will generally stay until 10pm or so. However, if no students arrive before 8pm then I will leave and find some other way to spend my evening. If you can only arrive after 8pm and are concerned that I will depart before you can get there, feel free to contact me in advance and let me know that you are coming.

**Texts:** Unfortunately, there is no true textbook for a course like this.

- The closest things available is the set of *Notes on Relativity and Cosmology* that I have prepared. You can pick up a black and white copy from the bookstore – it's called the PHY312 course reader. You can also download the (color) PDF file from the course website at: <http://physics.syr.edu/courses/PHY312.03Spring>. It's a big file though (300+ pages).
- We will also use *Relativity* by A. Einstein as a supplementary text. Each chapter of my notes indicates which chapters of Einstein you should be reading simultaneously. This will show you a second perspective on the subject, and I think you will enjoy seeing what Albert himself has to say.

In addition, a number of further supplemental texts are recommended below if you would like to investigate a particular topic more deeply. These may be especially useful for your course project.

### 3 Coursework and Grading

Let me begin by reminding you that this is an upper division ( $\geq 300$ ) course. This does *not* mean that the course is inaccessible to Freshmen and Sophomores but it *does* mean that students in this course will be treated as and expected to behave as mature learners: demonstrating initiative, asking questions, beginning and handing in homework promptly, and generally taking responsibility for their education. Correspondingly, the framework for this course will not be fixed as rigidly as it would be for any lower division course that I would teach.

The plan for this course is fairly simple. There will be weekly homework, two exams, and a project. The project will be worth 20% of the course grade, the two exams 30% each (making 60% together), and the homework will be worth 20%. Each assignment will receive a letter grade (A, A-, B+, etc.) and these grades will be averaged on a linear scale.

Note that the grading policy in this class may be different from that of other science/math courses you may have taken. While it is always difficult to state what is a ‘fair’ grading policy, I think it is clear what sorts of grading policies would *not* be reasonable for this course: *un-fair* grading policies would include

- i) grading on a strict curve, allowing only 25% of the class to get an “A” no matter how well they do,
- ii) grading on the ‘usual’ A=90-100, B=80-90, etc. scale – the material in this class is ‘harder’ than that.

You can be sure that neither of these will occur. When your first assignments are returned, you’ll be able to see more concretely what the grading scale in this course does look like.

**Homework:** The homework will be assigned weekly. In general, it will be assigned on a Thursday and will be due the following Thursday. I will warn you in advance if solving some homework problems will require material from the intervening Tuesday’s class. While I encourage you to work together on the homework, *the homework you turn in must represent your own understanding*. Homework that is simply copied will not be accepted.

I will hand out solutions to the homework on the day you turn it in. As a result, homework will be turned in on time at the beginning of class. If this is not possible due to some exceptional circumstances, it is your responsibility to contact me before class (as far in advance as possible) to discuss the matter.

Doing the homework will be an important part of learning the material in this course and I strongly recommend that you begin to work on it early and not leave the homework for the last minute. Especially because the topics we study are likely to be beyond your current intuition, you will need to devote some time and effort to working with these new ideas and building up a *new* kind of intuition. The homework is the primary means

through which you do this. I will, however, try to keep the workload reasonable for this course.

Problem sets will be graded ‘holistically,’ so that you will only receive one grade for each assignment (as opposed to grading each problem separately). However, I will write a number of comments on your papers and, in addition, I will provide detailed solutions to most homework problems.

By the way, you are more than welcome to work on the homework together, or to come and work on it at my office hours – *especially the one on Wednesday evening*. The reason that I hold that office hour in a different room is that I would like the room to be available for you to simply come and sit and work on your homework, either individually or in groups. You can ask me questions as they arise. You’re also welcome to meet people there to discuss the homework even if you don’t plan to ask me anything. Please feel free to consider this as a weekly study and/or chat session for Relativity, Cosmology, and related matters. We can turn on the teapot and have a nice place to work and discuss physics. This has been quite popular in the past, and many students participate. Note: Due to the unique nature of this course, the physics clinic is unlikely to be a useful resource for PHY312.

**Exams:** The first exam will be Thursday March 6, just before Spring Break.

*Note: You have now been warned of this date. Do **not** come to me with a story about how you bought a ticket to go somewhere for Spring Break and need to take the exam early so that you can leave.* The date of the second exam will depend on how things go, but will probably be Tuesday, April 22. The second exam will not be (explicitly) cumulative, though of course material from the last part of the course will build on material from the first part.

You should be aware that, despite the fact that there will be no cumulative final exam for this course, we *will* use the time assigned to PHY312 during finals week for you to display and present your course projects. This period is 5:00am -7:00pm, Tuesday May 6. The class time available for this course is far too short not to make use of every possible meeting time.

**The Project:** A significant part of your work in this class will be a project of your choice. The general guidelines are that the project should

- a) in some way demonstrate your understanding of topics covered in this course,
- b) be written in your own words,
- c) be about the same amount of work as a term paper (which I would consider to be about 10-20 pages in a reasonable type-style like this one) and

- d) include references to outside sources as appropriate for the project.

Projects will be graded on a combination of creativity, initiative, and the understanding of course material (and outside reading) that they demonstrate. The rule of thumb should be that your project should be something that you are proud of and not something that is quickly thrown together.

Each project must be approved by me, so you need to discuss your project with me before you begin. You are welcome to continue to consult with me while you work on your project. Each of you need to ‘contract’ with me for a project by March 25 (shortly after Spring Break). At this point, you and I will have more or less agreed on what your project will be. As a result, you should come and talk to me even earlier in order to bounce a few ideas around before making a final decision. It is a good idea to start thinking about your project early!!! You will be asked to give a (very) short presentation about your project at the end of the semester. **The projects are due at the start of the scheduled final exam period, 5pm on Tuesday, May 6.**

Some suggestions for projects are:

- i) A standard report/term paper on a topic related to relativity, gravity, or cosmology.
- ii) For web people: building a set of relativity/cosmology web pages either as an electronic ‘term paper’ or for educational purposes.
- iii) For any programmers out there: writing a small computer game (something like asteroids???) that treats relativity correctly. (Be careful here: *several* people have tried this in the past, but have bitten off more than they could really handle.) Another suggestion would be just writing a piece of code that demonstrates some important aspect of gravity, relativity, or cosmology.
- iv) For educators: preparing a set of instructional materials in relativity, cosmology, or related subjects for students at a level of your choosing.
- v) For the writing/journalism crowd: a short story, poem, or ‘magazine article’ dealing with some of the topics discussed in this course. Note: Poetry need not be especially good as literature – the limerick on the front page of this syllabus would be of sufficient quality. Poetry may be set to music (i.e., lyrics for a song, possibly ‘revised’ lyrics to an existing song).
- vi) For any artists: A work of visual, audio, or performing art could in principle be an excellent project. One year someone wrote a nice play. However, it must in some way involve your understanding of the course material. If you are interested in this kind of project, be extra sure to discuss it with me carefully.

- vii) Anything else you can think of that follows the basic guidelines given above. I'm quite flexible. If you think of an interesting project, I'll be happy to discuss it with you and see if we can make it work!!!

The project is an opportunity for you to get what *you* want out of this course and to extend the course in a direction that you would like it to go. I will therefore expect that your project will demonstrate either substantial creativity on your part or some further reading beyond what we will cover in class. This reading can be from books (such as those suggested in section 4) or magazines such as Scientific American, Physics World, or Physics Today. For example, you might wish to study one of the following topics in more detail:

- 1) black holes (their structure, collisions, or hawking radiation)
- 2) the big bang
- 3) higher dimensional theories of physics (Kaluza-Klein theories or 'braneworlds')
- 4) the history of relativity
- 5) experimental tests of relativity, or experiments in cosmology. These can be recent experiments or a historical treatment of the early experiments.
- 6) current and/or future NASA/ESA missions to study relativity and/or cosmology (HUBBLE, CHANDRA, other x-ray satellites, COBE, MAP, PLANCK, LISA, balloon experiments such as last year's Boomerang, and others)
- 7) the mathematical structure of general relativity
- 8) gravitational waves
- 9) gravitational lensing
- 10) Sagittarius A\* (the black hole at the center of our galaxy)
- 11) X-ray telescopes and their observations of black holes
- 12) LIGO
- 13) Current ideas about 'more fundamental' theories of gravity (loop gravity, string theory, noncommutative geometry).
- 14) Closed Timelike Curves (aka 'time machines')
- 15) Wormholes
- 16) The Global Positioning System

Feel free to talk with me about where and how to locate references on these and other subjects. Some examples of past course projects can be found on the PHY312 web page (see <http://physics.syr.edu/courses/PHY312.03Spring>). I encourage you to build from what other students have done in the past.

However, your challenge will be to go beyond their work and create something new.

*Warning:* not all of these projects received high grades. If you look through them, it will probably be clear to you which projects are the best. Feel free to discuss them with me if you would like.

*Additional Note:* Let me repeat that the project must be your own work. In particular, wholesale copying of entire sentences from your sources is not allowed.

### 3.1 Creating your Project

Following the steps below will help you to create a successful course project.

1. (late Feb. or early March) Begin to think about what you would like to do. Read through the suggestions above and leaf through some of the suggestions for further reading below. Come and talk to me in order to bounce around a few ideas.
2. (by March 25) Settle on a rough plan and have me approve it. This will be our ‘contract.’
3. Read outside sources and begin creative work, consulting with me as appropriate. Prepare a rough draft, perhaps showing it to me. Consult the checklist below during this process.
4. Prepare the final version and have it ready to turn in by 5pm on Tuesday May 6. Again, consult the checklist below to make sure that your project is complete.
5. Be ready to give a *brief* (5 minute or less) summary of your project at 5:00pm on Tuesday, May 6.

### 3.2 Project Checklist

The checklist below will help you to be sure that your project is complete. Remember though, the important thing is that you discuss your project with me and that we agree on what items will be included. The checklist below is not a replacement for my commentary, it is merely a tool to help you interpret my comments and expectations.

Your project should:

1. Include an introduction which a) provides a general overview and b) shows how your project is related to relativity and/or cosmology: Make sure that you describe the connection of your project to these themes. You should do this even if you feel that the connection is completely obvious, though the description can be brief. You and I should agree on the connection in our contract.

2. Be well-defined and focused: Your project should have clearly stated goals and a declared target audience. Is your project an attempt to explain something to the general public, or is it a deep study of some issue? We should agree on this in our contract.
3. Be thorough: Your project should take the time to explore material in some depth. It must reach beyond what we have done in class through the use of your own creativity and/or through the study of additional sources. Again, we should discuss this in our contract. See 8 below if your main project is a poster.
4. Illustrate your understanding of course material: The project must be tied to some of the material discussed in lecture. Your project should show that you understand this material well.
5. Be original: Clearly the project should represent your own work, but in fact I mean more than this. Many of the past course projects are available on the PHY312 web page for your exploration. Your goal should be to add to the set of such projects by doing something new. This may mean doing a different sort of project than has been done before or it may mean improving or extending a previous project.
6. Be well referenced: As a rule of thumb, a project should draw from *at least* 5 outside references. *Also*: Be sure to cite a reference for each separate fact you use from it.
7. Include proper citations: Individual facts, tables, arguments, etc. should be referenced with the source where you found them. Any graphics which are not of your own making should also be individually referenced. You may use either footnotes or endnotes. It is not sufficient to simply put a bibliography at the end.
8. Include something to display on Tuesday, May 6 and something to turn in: I would like you to have something to show the other students which is not just a typed paper. If your main project is a paper, this might be a small poster (the size of a large piece of paper) listing the highlights and including a graphic or two. You should also have something to turn in which shows the depth of your research. For example, if your main project is a poster, you will be able to fit very little on the poster itself. In this case, please also submit a list of “things I wanted to include that would not fit on my poster.” Each item on this list should have a short description, at least a couple of sentences long. *I will also want your original project*, including relevant computer files for web pages, computer programs, etc.
9. Be complete: In particular, if you design a web site or write a computer program, be sure that it is actually working before the due date!

## 4 Some Suggestions for Further Reading

Einstein's book *Relativity* covers only the very basics of relativity and cosmology. We will do quite a bit more in this course, and the lecture will be your main source of information for this extra material. This means that, together with the notes that I have prepared, your notes from class will be a primary source of study material. We will work with a number of complicated 'space-time diagrams' and, while this will require no artistic ability whatsoever, I do recommend that you purchase a set of colored pens or pencils and bring them to class.

Unfortunately, it is difficult to find a single book which includes the right balance of material and is written on the right level for this course. I have chosen Einstein's *Relativity* and the notes that I have prepared as the main texts. A list of suggestions for further reading can be found below. Each of those books does *more* than we will cover in this course for some topics but *less* for others. These books will mainly be helpful if you would like to learn more about some particular aspect of relativity or cosmology, to satisfy your personal curiosity and/or for your course project. The following books are on reserve in the Physics Library (on the second floor of the Physics building). If you like them, you may want to order your own copies through the bookstore, Barnes and Nobles, or amazon.com. In my experience, all of these places charge the same price except that mail order places will charge you a few dollars for shipping. However, mail order services may be a few days faster in getting the book and tend to be more efficient than the SU bookstore.

1. *Cosmology: The Science of the Universe* by Edward Harrison (New York, Cambridge University Press, 1981). This book is written from a less mathematical and more observational perspective than the lectures in PHY312. It gives an excellent treatment of many aspects of cosmology from galaxies and quasars to the cosmic microwave background.
2. *The cosmic frontiers of general relativity* (Boston, Little, Brown, 1971) by W. Kauffman. Kauffman gives a very nice discussion of many topics in or affected by GR such as Black Holes (this part is very thorough but still quite readable!), gravitational lenses, gravitational waves, active galactic nuclei, stars, and neutron-stars.
3. *Flat and Curved Space-times* by George Ellis and Ruth Williams (New York, Oxford University Press, 1988). This is an excellent general book on special and general relativity for those who would like to see more mathematics than we will do in this course. In particular, it has a very nice treatment of black holes.
4. *Principles of Cosmology and Gravitation* by Michael V. Berry (New York, Cambridge U. Press, 1976). Berry's book provides a somewhat more technical treatment of General Relativity than we will experience in this course.

While it is less mathematical than that of Ellis and Williams, Berry does assume a certain familiarity with many concepts of physics. In his own words, the aim of the book is “to present a theoretical framework powerful enough to enable important cosmological formulae to be derived and numerical calculations to be performed.” A down side of this book is that it is a bit old.

5. *Discovering Relativity for yourself, with some help from Sam Lilley* by Sam Lilley (New York, Cambridge University Press, 1981). This is an excellent book on the basics of special and general relativity, but it does not cover any advanced topics. It may be a good resource for working homework problems in this course.
6. *A traveler's Guide to Spacetime: An Introduction to the Special Theory of Relativity* by Thomas A. Moore (New York, McGraw-Hill Inc., 1995). This lovely book gives a good, modern, introduction to special relativity and may be helpful for someone who would like to read another discussion of this material. Unfortunately, it does not address accelerated reference frames or General Relativity.
7. *Special Relativity* by Anthony French (New York, Norton, 1968). This is an excellent traditional book on special relativity that will be of interest for those with more physics background. It has some very nice discussions of the experiments related to special relativity but readers should be prepared for a fair amount of algebra.
8. *Time, Space, and Things* by B.K. Ridley (New York, Cambridge University Press, 1984). This is a nice book about special relativity and particle physics which may be useful if you want a very non-technical presentation.
9. *The Ethereal Aether* by L.S. Swenson (Austin, University of Texas Press, 1972). This is a beautiful book about the history of the study of light, and especially of the experiments on the nature, velocity, etc. of light. It is not a relativity book per se, but it discusses a lot of the history of relativity as it discusses the relevant experiments. This includes not only the pre-Einstein history, but also the story of what happened during the time between when Einstein published his work and when it became fully accepted.
10. *Geometry, Relativity, and the Fourth Dimensions* by R. v.B. Rucker, (Dover, New York, 1977). This is a nice book that concentrates on the geometrical aspects of curved space and of four-dimensional spacetime, both flat and curved. It is easier to digest than many of the books above.
11. *Theory and experiment in gravitational physics* by Clifford M. Will, (Cambridge University Press, New York, 1993) QC178.W47 1993. This is the definitive work on experimental tests of General Relativity. A less technical version of the same work is: *Was Einstein Right? putting General Relativity to the test* by Clifford M. Will (Basic Books, New York, 1993).

## Course Calendar

The following is a tentative calendar for PHY 312, Spring 2001.

Week 1: (1/14) Background material on pre-relativistic physics (coordinate systems, reference frames, the Newtonian assumptions about time and space, inertial frames, begin Newton's laws).

Week 2: (1/21) Newtonian mechanics: inertial frames and Newton's laws. Electricity, magnetism and waves. The constancy of the speed of light. The ether and the Michelson-Morely experiment.

Week 3: (1/28) The postulates of relativity. What do the postulates of relativity imply? Spacetime diagrams, simultaneity, light cones, and time dilation.

Week 4: (2/4) More work with spacetime diagrams: length contraction and more subtle problems – the train 'paradox,' the interval, proper time and proper distance, a bit on Minkowskian geometry, the twin 'paradox.'

Week 5: (2/11) More on Minkowskian geometry. Begin acceleration. The head-light effect.

Week 6: (2/18) Acceleration in special relativity.

Week 7: (2/25) Dynamics (forces, energy, momentum, and  $E = mc^2$ ).

Week 8: (3/4) Gravity, light, time, and the local equivalence principle. Exam 1.

Week 9: (3/11) *SPRING BREAK*

Week 10: (3/18) Nonlocal calculations in GR. Time dilation and GPS.

Week 11: (3/25) Curved spaces and curved spacetime.

Week 12: (4/1) The Metric: the mathematical description of a curved surface.

Week 13: (4/8) The Einstein equations and the Schwarzschild solution. The classic tests of GR: Mercury's orbit, the bending of light, and radar time delays. Begin black holes.

Week 14: (4/15) More on Black Holes: inside, outside, etc.

Week 15: (4/22) Second exam. A little cosmology.

Week 16: (4/29) More cosmology. If time permits, we may discuss compact universes, closed timelike curves, the periodic Milne Universe. Kaluza-Klein, higher dimensions, other extensions of Einstein's theories.

Brief Project Presentations: 5pm Tuesday, May 6.