List of project topics:

1. Modify your molecular dynamics code to model the part of the Solar System including Earth, Jupiter and the Sun. Show that the Earth's orbit is unstable if placed in a orbit which brings it close to Jupiter. Use the simple Newtonian inverse square law for the gravitational force. Use realistic values for the planets' orbital parameters and masses.

2. Modify your molecular dynamics code to study the leading relativistic corrections to Newton's law of gravity. To the usual inverse square gravitational force add a term $\frac{\alpha}{r^4}$. Consider just the Sun- Mercury system. Place Mercury in an *elliptical* orbit and using your simulation show that the entire orbit rotates in space. This is called *precession.* Compute the rate of rotation of the resulting elliptical orbit as a function of alpha and plot your results. What is the physical value of α ?

3. The logistic map in a chaotic regime. Measure quantitatively the rate of divergence of two trajectories differing only slightly in their initial conditions in a chaotic regime. This divergence is approximately exponential in time $|\Delta x| \sim e^{Lt}$ where L is termed the Lyapunov exponent. Measure the latter for the logistic map by averaging data for several different runs. Modify your code to allow for a zooming feature. Hence show that the values of x lie on a fractal which possesses structure at all length scales.

4. Thermodynamics from molecular dynamics. Modify the molecular dynamics code to measure the average temperature and pressure for a set of N molecules on a box. Take $N = 10, 20$. The temperature is defined as the mean kinetic energy carried by the molecules and the pressure is related to the (absolute) value of the total change in momentum resulting from collisions with the walls per unit of time.

   $$P = \frac{1}{A\tau}\Delta p$$

   where A is the area of a side of the box and τ is the time elapsed. It may be useful to generate plots of these quantities as a function of time. Show that $P \sim T$ for high temperatures.

5. A simple generalization of the Ising model discussed in class is the q-state Potts model which is composed of integer spins which can take one of q possible states $s_i = 0, \dots q-1$ and has energy

   $$E = -J \sum_{<ij>} \delta(s_i s_j)$$

   where $\delta(s_i, s_j) = 0$ unless $s_i = s_j$ when $\delta = 1$.

   Using the simple Metropolis algorithm discussed in class simulate this model for $q = 3$ Your answer should include measurements of the magnetization, internal energy and specific heat as a function of the inverse temperature. Does the system undergo a phase transition ?