***PHY212 Lecture 7***

1)2006 exams/solutions is at http://physics.syr.edu/courses/PHY212.07Fall/Quizzes_Exams.htm
2)You may bring a formula sheet(written on one page only) to the exam (Exam1 Thursday Sept 20).

1. **Review mechanics**
   **N**et **F**orce (**F**) on point object of mass m/**a**cceleration(**a**) of object ;

   Newton's law(s) : $\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{V}}{dt}, \quad \mathbf{v} = \frac{d\mathbf{X}}{dt} \longrightarrow$

   work/KE relation : $W = \int_i^f \mathbf{F} \cdot \mathrm{d}\mathbf{x} = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

   In general, $\mathbf{F} = -\frac{\partial U}{\partial \mathbf{X}} +$ (**N**on-gradient **F**orce), $U =$ potential enery(PE).

2. **Some reasons** for introducing work, potential energy and potential:
   - They are **scalar** quantities and so easier to deal with.
   - These concepts may be intuitively closer to everyday communication.

3. **Gravitational force/field**:
   Force/PE : $\mathbf{F} = -\frac{\partial U}{\partial \mathbf{X}}$

   Field/Potential : $\mathbf{F}/m = -\frac{\partial (U/m)}{\partial \mathbf{X}} \longrightarrow \mathbf{g} = -\frac{\partial V}{\partial \mathbf{X}}$

   work/KE relation becomes $K_i + U_i = K_f + U_f =$ Total energy.

   This can also be written as $W = \Delta K = -\Delta U$

4. **Electric force/field** (Analogous to the gravitational force/field )
   Force/PE : $\mathbf{F}_e = -\frac{\partial U_e}{\partial \mathbf{X}}$ (defining relation for electrical PE)

   Field/Potential : $\mathbf{F}_e/q = -\frac{\partial (U_e/q)}{\partial \mathbf{X}} \longrightarrow \mathbf{E} = -\frac{\partial V_e}{\partial \mathbf{X}}$

   $V_e = U_e/q \quad \Rightarrow \quad$ units = $J/C$=volts(V).

   - Acceleration of charges by a potential difference between two parallel plates: Use $W = \Delta K = -\Delta U$.

5. **Calculating the electric potential($V$) from the electric field (E)**.
   First find **E** (e.g. by Gauss' law) then
   $\mathbf{E} = -\frac{\partial V_e}{\partial \mathbf{X}} \longrightarrow$

$$V(\mathbf{x}) - V(\mathbf{x}_{ref}) = -\int_{\mathbf{X}_{ref}}^{\mathbf{X}} \mathbf{E}(\mathbf{x}') \cdot d\mathbf{x}'$$

For a point charge Q located at $\mathbf{x}_Q$, $\mathbf{E}(\mathbf{x}) = \frac{kQ}{r^2}\hat{\mathbf{r}}$, $r = |\mathbf{x} - \mathbf{x}_Q|$, $k = \frac{1}{4\pi\varepsilon_0}$.

$V(\mathbf{x}) - V(\mathbf{x}_{ref}) = -\int_{\mathbf{x}_{ref}}^{\mathbf{x}} \frac{kQ}{r'^2} dr' = \frac{kQ}{r} - \frac{kQ}{r_{ref}}$, $r = |\mathbf{x} - \mathbf{x}_Q|$.

If we take the reference point $\mathbf{x}_{ref}$ to be at $\infty$ where $\lim_{r\to\infty} \frac{kQ}{r} = 0$, then $V(\mathbf{x}) - V(\infty) = \frac{kQ}{r}$.

Therefore, relative to a point at $\infty$, the electric potential $(V)$ at a point $\mathbf{x}$ due to a point charge $Q$ located at $\mathbf{x}_Q$ is

$V(\mathbf{x}) = \frac{kQ}{r}$, $r = |\mathbf{x} - \mathbf{x}_Q|$.

6. **Calculating the electric potential($V$) of a charge distribution directly from the electric potential of a point charge**.

   - $V$ at a position $\mathbf{x}$ due to a point charge $Q$ located at a different position $\mathbf{x}_Q$ is defined to be
     $V(\mathbf{x}) = \frac{kQ}{r}$, $r = |\mathbf{x} - \mathbf{x}_1|$.
     The electric potential energy of another point charge q (call it a test charge) now placed at position $\mathbf{x}_q$ is
     $U = qV(\mathbf{x}_q) = \frac{kqQ}{r}$, $r = |\mathbf{x}_q - \mathbf{x}_Q|$.
   - $V$ at position $\mathbf{x}$ due to a **collection of point charges** $Q_1,\ Q_2,\ ...$ located at respective positions $\mathbf{x}_1,\ \mathbf{x}_2,\ ...$ is given by

     $$V(\mathbf{x}) = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + ... = \sum_i \frac{kQ_i}{r_i}, \qquad r_i = |\mathbf{x} - \mathbf{x}_i|.$$

     Again, the electric PE of another point charge q now placed at position $\mathbf{x}_q$ is
     $U = qV(\mathbf{x}_q) = kq\sum_i \frac{Q_i}{r_i}$, $r_i = |\mathbf{x}_q - \mathbf{x}_i|$.
   - $V$ at the point $\mathbf{x}$ due to a **continuous charge distribution**, which is contained in a certain region $R$ in space, is given by

     $$V(\mathbf{x}) = \int_R \frac{kdQ}{r}, \qquad r = |\mathbf{x} - \mathbf{x}_{dQ}|.$$

     where the charge element $dQ$ is treated like a point charge. The electric PE of a point charge $q$ place at position $\mathbf{x}_q$ is

     $$U = qV(\mathbf{x}_q) = kq\int_R \frac{dQ}{r}, \qquad r = |\mathbf{x}_q - \mathbf{x}_{dQ}|.$$

   - Finally, just to be complete, the electric PE of a collection of point charges $Q_1,\ Q_2,\ ...$ located at respective positions $\mathbf{x}_1,\ \mathbf{x}_2,\ ...$ is given by

     $$U = \frac{1}{2}\sum_{i\neq j} \frac{kQ_iQ_j}{r_{ij}}, \qquad r_{ij} = |\mathbf{x}_i - \mathbf{x}_j|.$$

     Similarly, the electrical PE of a continuous charge distribution occupying some region R of space is

$$U = \frac{1}{2}\int_R \frac{kdQ_1 dQ_2}{r_{12}}, \qquad r_{12} = |\mathbf{x}_{dQ_1} - \mathbf{x}_{dQ_2}|.$$

7. **Equipotential regions**:

   An equipotential region is a region of space in which the electric potential is the same at every point inside that region.

   That is, $V(\mathbf{x}) = \text{constant}$ in an equipotential region.

   The electric field in the core of the equipotential region is zero. Therefore electric field lines must be normal(perpendicular) to equipotential surfaces and equipotential lines.

   One can have equipotential lines, equipotential surfaces, equipotential volumes etc. A region of space occupied by a **conductor** is an equipotential region.

8. **Electrostatic shielding**:

   A hollow region inside a conducting material(equipotential region) cannot be reached by electric field lines from the outside. Therefore something placed inside the hollow is protected from potentially harmful external electric fields.

9. **Examples**:

   - Potential of **charge distributions**(ring, line charge, uniformly charged sphere).
   - Electric PE (U) of an **electric dipole** in a uniform external field **E**.

     $\mathbf{F}_e = -\frac{\partial U_e}{\partial \mathbf{x}} \quad \longrightarrow$

     $$U(\mathbf{x}) - U(\mathbf{x}_{ref}) = -\int_{\mathbf{x}_{ref}}^{\mathbf{x}} \mathbf{F}(\mathbf{x}')\cdot d\mathbf{x}'$$

     Consider a dipole consisting of the two charges $+|q|$ and $-|q|$ located at $\mathbf{x}_+$ and $\mathbf{x}_-$ respectively. Then

     $$U = U(\mathbf{x}_+) + U(\mathbf{x}_-) = -\int_{\mathbf{x}_\infty}^{\mathbf{x}_+} \mathbf{F}(\mathbf{x}')\cdot d\mathbf{x}' - \int_{\mathbf{x}_\infty}^{\mathbf{x}_-} \mathbf{F}(\mathbf{x}')\cdot d\mathbf{x}'$$

     $$= -|q|\mathbf{E}\cdot\int_{\mathbf{x}_\infty}^{\mathbf{x}_+} d\mathbf{x}' - (-|q|)\mathbf{E}\cdot\int_{\mathbf{x}_\infty}^{\mathbf{x}_-} d\mathbf{x}'$$

     $$= -|q|\mathbf{E}\cdot(\mathbf{x}_+ - \mathbf{x}_\infty) + |q|\mathbf{E}\cdot(\mathbf{x}_- - \mathbf{x}_\infty) = -|q|\mathbf{E}\cdot(\mathbf{x}_+ - \mathbf{x}_-)$$

     That is, $U = -\mathbf{p}\cdot\mathbf{E}$

     Compare this with torque about a given point $\mathbf{x}$ in space

     $$\vec{\tau} = \vec{\tau}_+ + \vec{\tau}_- = \mathbf{r}_+ \times |q|\mathbf{E} + \mathbf{r}_- \times (-|q|)\mathbf{E} = |q|(\mathbf{r}_+ - \mathbf{r}_-)\times\mathbf{E}$$

     $$\mathbf{r}_+ = \mathbf{x}_+ - \mathbf{x}, \qquad \mathbf{r}_- = \mathbf{x}_- - \mathbf{x}$$

     $$\vec{\tau} = \mathbf{p}\times\mathbf{E}.$$