

I have done this work by myself worked out copy.

Time of Workshop _____

Check name of Workshop instructor:

Suphatra Adulrattananwat

Earnest Akofof

Michele Fontanini

Pramod Padmanabhan

Eric West

Some useful constants:

Magnitude of electron and proton charge $e = 1.60 \times 10^{-19} \text{ C}$

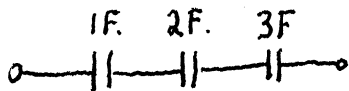
Electric force constant $1/(4\pi \epsilon_0) = 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$

Gravity force constant $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$

Electron mass $m_e = 9.1 \times 10^{-31} \text{ kg}$

1 (20 points) Some short questions.

- a) As shown, 1 F, 2 F and 3 F capacitors are connected in series. What is the net capacitance of this combination?



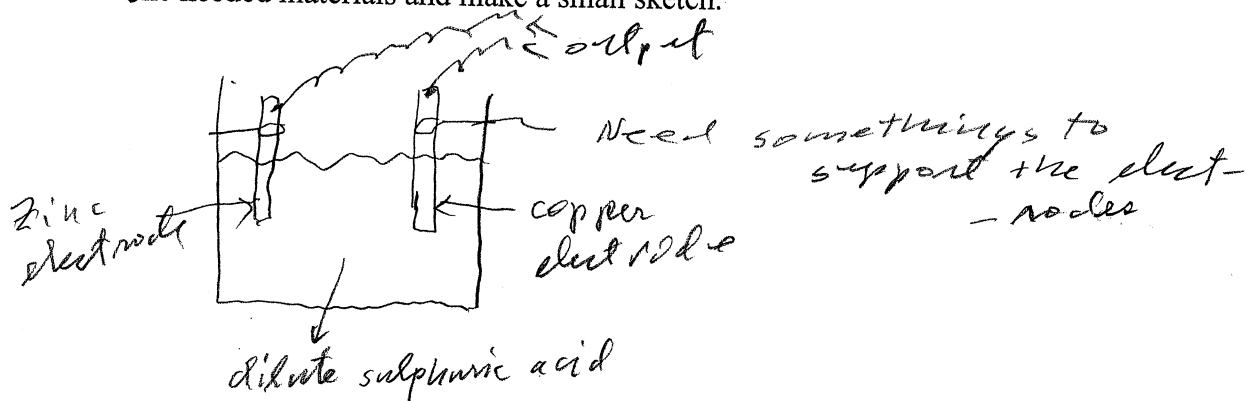
$$\frac{1}{C} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

$$C \approx 0.55 \text{ F.}$$

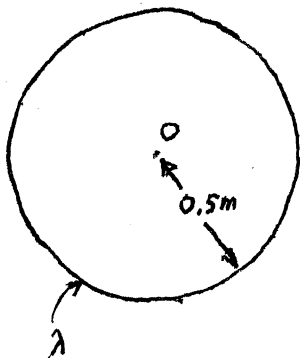
- b) A copper wire 50 meters long has a cross sectional radius of 2×10^{-3} m. Using the resistivity, ρ of copper as $1.77 \times 10^{-8} \Omega \text{ m}$, find the electrical resistance of this wire.

$$R = \frac{(1.77 \times 10^{-8})(50)}{\pi (4 \times 10^{-6})} \approx 0.0704 \Omega$$

- c) Describe how you could construct a small electric battery. Specify the needed materials and make a small sketch.



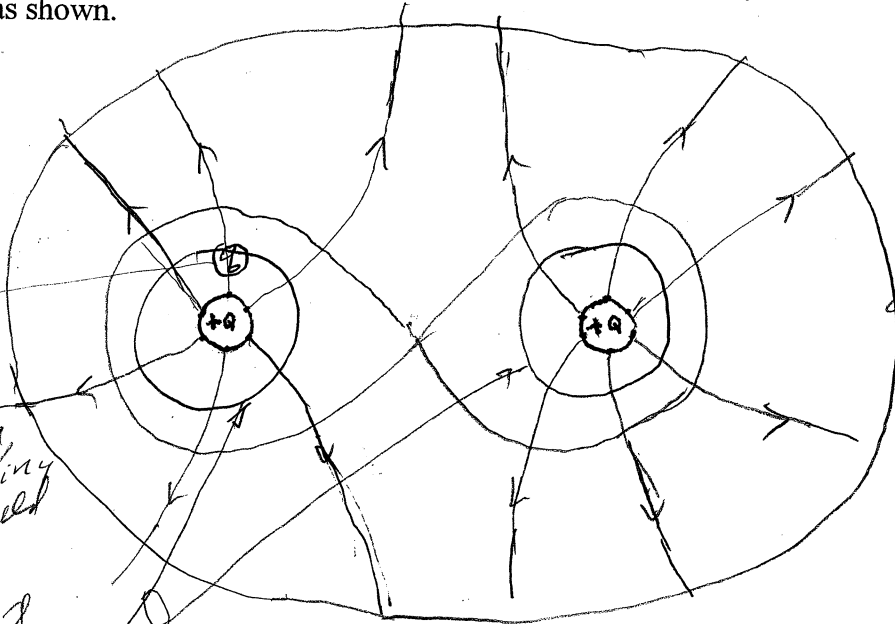
- d) Positive electric charge density, $\lambda = 3 \times 10^{-6}$ C/m is spread uniformly around the circumference of a circle of radius 0.5 m. What is the electric potential, V at the center, O of this circle?



$$\begin{aligned}
 V_{at\ O} &= \left(\frac{1}{4\pi\epsilon_0} \right) \left(\frac{2\pi(0.5)}{0.5} \right) (3 \times 10^{-6}) \\
 &\approx 9 \times 10^9 (2\pi) (3 \times 10^{-6}) \\
 &\approx 1.7 \times 10^5 \text{ Volts}
 \end{aligned}$$

2) (20 points)

a) (10 points) Two positive point-like charges are separated by some distance as shown.



② would "like" to move to the lower potential region starting by moving along the field line shown

Highest equipotential line sketched

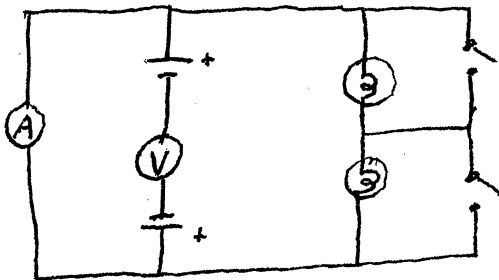
lowest equipotential curve sketched

Sketch the electric field lines for this arrangement, including arrows showing their appropriate directions. For definiteness take 6 representative field lines to emerge from each charge.

Next, sketch three different equipotential lines at different characteristic distances from the charges. Which of the ones you sketched has the highest potential?

Suppose a small positive charge, q is placed at rest on the highest equipotential. How will it move?

- b) (10 points) In the first circuits lab a student is given two 1.5 Volt batteries, two switches and two flashlight bulbs, each requiring 3 Volts for their proper operation. In addition he is given a voltmeter $-V-$ and an ammeter $-A-$ to monitor the voltage and current being supplied by the batteries. The switches should be able to turn on or off either bulb. His initial circuit looks like:

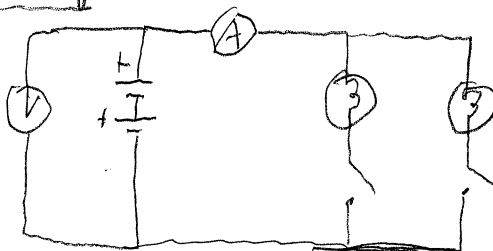


This initial attempt is not quite right. List the mistakes and show the correct wiring arrangement.

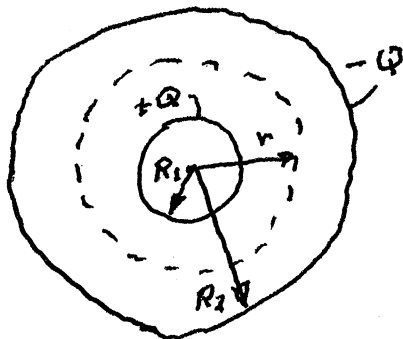
mistakes

- batteries reversed so their potential sums to zero instead of 3V.
- (A) should be in series with the two batteries.
- (V) should be across the batteries.
- The light bulbs should ^{be} in parallel rather than in series.
- The switches should ^{be} in series (rather than parallel) with their light bulbs.

correct wiring



- 3 (20 points) A particular capacitor consists of an internal spherical metal shell of radius R_1 and an external spherical metal shell of radius R_2 . A total positive charge Q is placed on the internal shell and a total charge $-Q$ on the external shell. The situation is illustrated below:



- a) (7 points) Use Gauss's Law with the help of a spherical Gaussian surface of radius r , in between R_1 and R_2 , to find the electric field $E(r)$. Express the result in terms of the symbols Q and ϵ_0 .

$$4\pi r^2 E(r) = \frac{1}{\epsilon_0} Q$$

$$E(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

b) (7 points) Find the difference in electric potentials at R_2 and R_1 , $V(R_2) - V(R_1)$, by integrating the formula

$$dV = -E dr.$$

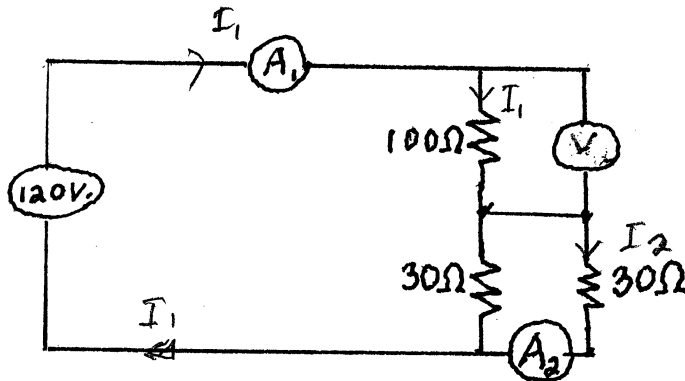
$$\int_{V(R_1)}^{V(R_2)} dV = - \frac{Q}{4\pi\epsilon_0} \int_{R_1}^{R_2} \frac{dr}{r^2}$$
$$= - \frac{1}{r} \Big|_{R_1}^{R_2}$$

$$V(R_2) - V(R_1) = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R_2} - \frac{1}{R_1} \right)$$

c) (6 points) Find the capacitance of this capacitor, using the basic definition of capacitance.

$$C = \frac{|Q|}{|V(R_2) - V(R_1)|} =$$
$$= 4\pi\epsilon_0 \left(\frac{1}{R_1} - \frac{1}{R_2} \right)^{-1} =$$
$$= 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$

4. (20 points) The circuit shown has three resistors connected to a 120 Volt source. A_1 and A_2 are ammeters while V is a voltmeter. The meters do not affect the operation of the circuit.



a) What is the effective resistance of all three resistors considered together?

$$100 + \frac{(30)(30)}{30+30} = 100 + 15 = 115 \Omega$$

b) What are the readings of ammeters A_1 and A_2 ?

$$I_1 = \frac{120}{115} \approx 1.04 \text{ Amperes}$$

$$I_2 = \frac{1}{2} I_1 \approx 0.52 \text{ Amperes}$$

c) What is the reading of the voltmeter and how much power is being dissipated in the 100Ω resistor?

$$V = (I_1)(100) \approx (1.04)(100) \approx 104 \text{ Volts}$$

$$\text{Power in } 100\Omega \text{ resistor} = (I_1)^2(100) \approx 108 \text{ Watts}$$

d) How much power is the voltage source supplying to the circuit? If the cost of electricity is 10 cents per kilowatt-hour, what is the cost of operating the circuit for a full day?

$$\text{Total Power supplied by battery} \approx (120 \text{ Volts}) \times (1.04 \text{ Amps})$$

$$\approx 125 \text{ Watts}$$

$$= 0.125 \text{ kW}$$

Operating the circuit for 24 hours would cost

$$(24 \times 0.125) \text{ kWh} \times \$0.1 = \$0.3 = 30 \text{ cents}$$

5) (20 points) In this problem it will be convenient to remember:

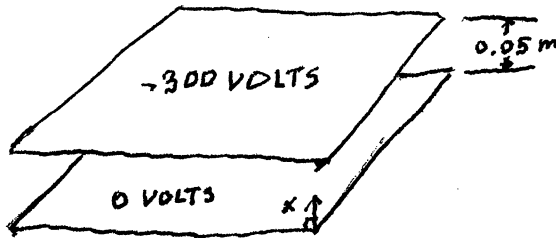
$$e = |\text{electron charge}| = 1.602 \times 10^{-19} \text{ Coulombs,}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ Joules.}$$

$$m_e = \text{electron mass} = 9.11 \times 10^{-31} \text{ kg}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$$

Two large metal plates, each with area 0.3 m^2 , are separated by a distance 0.05 m . The lower plate has a potential $V(0) = 0 \text{ Volts}$ while the upper plate has a potential $V(0.05) = -300 \text{ Volts}$. Let x be the perpendicular distance measured up from the lower plate.



a) Assuming that edge effects are negligible so that the electric field, E is constant everywhere between the two plates and zero elsewhere, what is $V(x)$, the potential as a function of x ?

Since

$$E = -\frac{dV}{dx}$$

and $E = \text{constant}$ we have

$V = c x$ since \rightarrow to be determined

$$V = 0 \text{ at } x = 0.$$

$$\text{At } x = 0.05 \text{ m, } V = -300 \text{ V.}$$

$$c(0.05) = -300, \quad c = \frac{-300}{0.05} = -6000 \Rightarrow \boxed{V(x) = -6000x}$$

b) What is the constant electric field between the two plates? In which direction does it point?

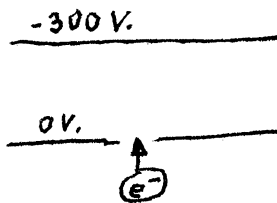
$$E = -\frac{dV}{dx} = 6000 \frac{\text{Volts}}{\text{meter}}$$

The vector \vec{E} points straight up.

c) What is the total field energy due to the electric field between the two plates?

$$\begin{aligned} \text{Field energy} &= \left(\frac{1}{2} \epsilon_0 |E|^2 \right) \times \text{volume} \\ &= \frac{1}{2} (8.85 \times 10^{-12}) (36 \times 10^6) (0.3)(0.05) \\ &= 2.39 \times 10^{-6} \text{ Joules} \end{aligned}$$

d) Now a small hole is made in the lower plate and an electron with a kinetic energy of 10 eV is shot straight up through the hole as shown. How far up will the electron travel before falling back down again?



The KE of the electron will be used up when the electron gains 10 eV of potential energy. This happens when the electron goes through a potential difference of 10 volts or $\frac{10}{300} = \frac{1}{30}$ of the way to the top; namely $\frac{1}{30} \times (0.05) \approx \boxed{0.0017 \text{ meters}}$

e) What was the velocity of the electron when it first entered the hole?

$$10 \text{ (eV)} \times \frac{1.6 \times 10^{-19} \text{ Joules}}{1 \text{ (eV)}} = 1.6 \times 10^{-18} \text{ Joules} = \frac{1}{2} m_e v^2$$

$$v^2 = \frac{3.2 \times 10^{-18}}{9.11 \times 10^{-31}} = 0.35 \times 10^{13} = 3.5 \times 10^{12}$$

$$v = 1.87 \times 10^6 \frac{\text{m}}{\text{s}}$$

