

I have done this work by myself Worked out Copy.

Time of Workshop _____

Check name of Workshop instructor:

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Some useful constants:

Magnitude of electron and proton charge $e = 1.60 \times 10^{-19} \text{ C}$

Electric force constant $1/(4\pi \epsilon_0) = 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$

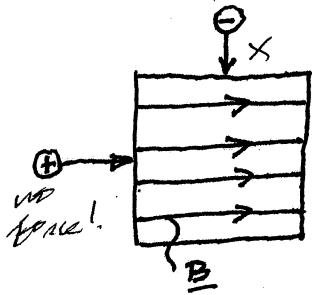
Magnetic force constant $\mu_0/(4\pi) = 10^{-7} \text{ T}\cdot\text{m}/\text{A}$

Gravity force constant $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$

Electron mass $m_e = 9.1 \times 10^{-31} \text{ kg}$

1(30 points) Some short questions.

a) Charged particles, with velocities and charges as shown are traveling into the uniform magnetic field shown. Sketch the initial direction of the magnetic force that acts on each particle.



b) An alternating electric current has the form:

$$I(t) = I_0 \sin(5t),$$

where t = time in seconds.

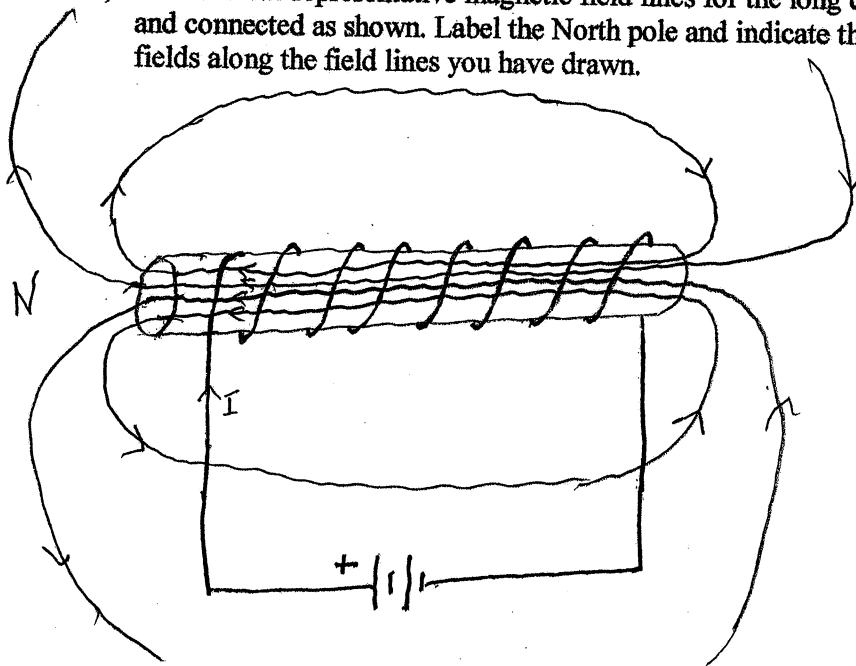
What is the frequency of this current?

$$5t = 2\pi, \quad \nu = \frac{1}{T} = \frac{5}{2\pi} \approx 0.8 \text{ Hz}$$

- c) At a party, one of your friends tells you that the magnetic field of the Earth is expected to flip in the future so that the Earth's North magnetic pole will coincide with the North geographical pole. Is he speaking the truth or has he had too much to drink? Explain.

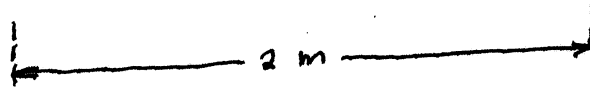
Geological evidence (at the sea floor) indicates that the magnetic field has often flipped in the past, with a typical interval of about 10^6 years. The source of the magnetic field is supposed to be currents in the Earth's molten iron core.

- d) Sketch some representative magnetic field lines for the long coil of wire, wound and connected as shown. Label the North pole and indicate the directions of the fields along the field lines you have drawn.



- e) Long wire 1 is carrying 3 Amperes into the page while long wire 2 is carrying 4.5 Amperes out of the page. What is the approximate magnitude and the direction of the magnetic field at point P, 2 meters away from the wires?

Net $I = 1.5 A$
 (1) (2) - out of page



$$\int \underline{B} \cdot d\underline{r} = \mu_0 I_{NET}$$

$$B \cdot (2\pi R) = (4\pi \times 10^{-7}) (1.5)$$

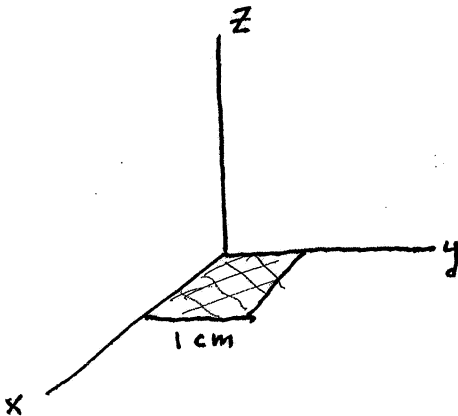
$$B = 1.5 \times 10^{-7} T$$

(direction: up as shown)

- f) A square with sides 1 cm in length is oriented as shown in the x-y plane. A uniform magnetic field:

$$\underline{B} = 0.1 \underline{i} + 0.2 \underline{j} + 0.3 \underline{k} \quad \text{Tesla}$$

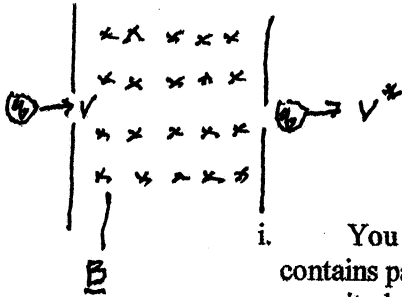
exists throughout the whole region. What is the magnetic flux passing through the square?



$$\begin{aligned} \Phi_{mag} &= \underline{B} \cdot \underline{A} \\ &= B \cdot (-0.1) \underline{k} \\ &= (0.3)(10^{-4}) \underline{k} \cdot \underline{k} \\ &= 3 \times 10^{-5} T \cdot m^2 \end{aligned}$$

2 (35 points)

- a) (15 points) As shown, a beam of particles with mass m and positive charge q is entering the region of interest through the slit on the left. The region of interest contains a magnetic field B oriented as indicated. The incoming beam contains particles with all different horizontal speeds v .



- i. You want to obtain a beam which emerges from the right hand slit and contains particles all having the same specified speed v^* . What is the magnitude and direction of an electric field you can apply in the region of interest to obtain this result? Give your answer in terms of the symbols provided. What is the kinetic energy of the outgoing particle?

The B field exerts an up force on q . To cancel it E must be pointing down and

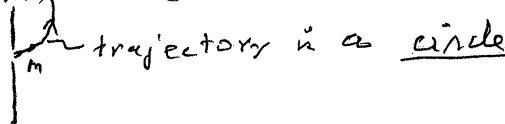
$qE = qv^*B$ or $E = v^*B$. The kinetic energy of q when it is outgoing is $\frac{1}{2}mv^{*2} = \frac{1}{2}m\left(\frac{E}{B}\right)^2$

- ii. How would you modify the electric field to obtain the same result when q is negative?

No modification needed. (q cancels out)

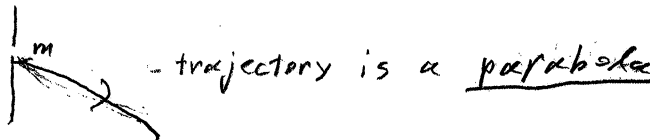
- iii. Sketch the trajectory of the particle if just the electric field were to be turned off. What is its geometric shape?

If E is turned off



- iv. Sketch the trajectory of the particle if just the magnetic field were to be turned off. What is its geometric shape?

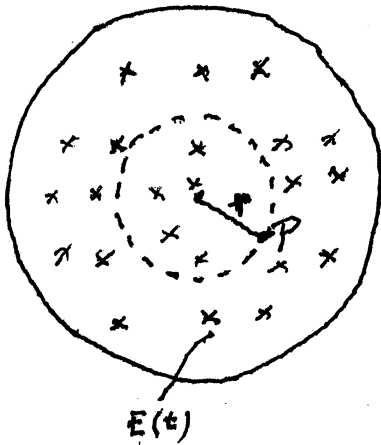
If B is turned off



- b) (10 points) In a region of space between two circular capacitor plates (shown in top view) there is a uniform electric field varying in time, as the capacitor charges up, according to:

$$E(t) = E_0(1 - e^{-\alpha t}),$$

where E_0 and α are constants. Using the Ampere-Maxwell equation, what are the magnitude and direction of the induced magnetic field at an arbitrary point P located at distance r from the center, at time t ? Give your answer in terms of the symbols provided.



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \vec{I} + \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

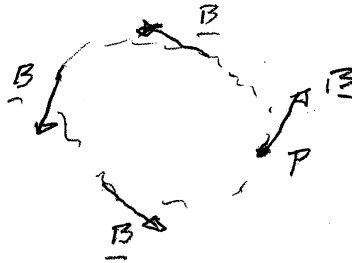
\downarrow
 $\mu_0 \epsilon_0$ with bound, π

$$2\pi r B = \mu_0 \epsilon_0 \pi r^2 \frac{dE}{dt}$$

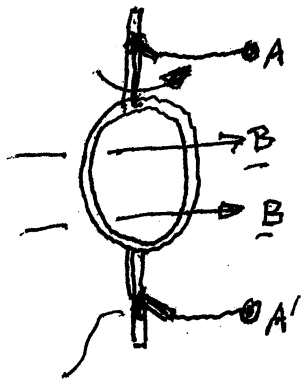
$$2\pi r B = \mu_0 \epsilon_0 \pi r^2 \alpha E_0 e^{-\alpha t}$$

$$B = \frac{\mu_0 \epsilon_0 \alpha E_0}{2} e^{-\alpha t}$$

direction is tangential:



- c) (10 points). An electric generator consists of 100 turns of wire wrapped around a frame of area 0.1 m^2 , rotating at an angular velocity of 10 radians per second about a symmetry axis perpendicular to a magnetic field of magnitude 0.5 T , as shown. What is the generated emf at time t (where $t=0$ corresponds to the time when maximum magnetic flux is piercing the coil)?



SLIP RING
(SLIDING CONTACT)

AA' → OUTPUT
CONNECTIONS

$$\oint \underline{E} \cdot d\underline{l} = - \frac{d}{dt} \Phi_{\text{mag}}$$

$$\begin{aligned} \Phi_{\text{mag}} &= (100)(0.1)(0.5) \cos(10t) \\ &= 5 \cos(10t) \end{aligned}$$

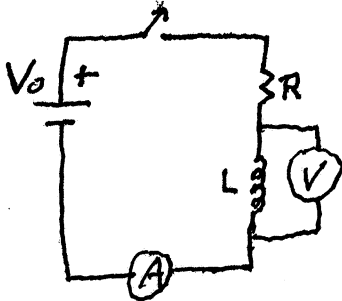
$$\text{emf} = - \frac{d}{dt} \Phi_{\text{mag}} = 50 \sin(10t) \text{ Volts}$$

3 (35 points).

a) (5 points) An 0.05 H inductor is connected in series with an 0.20 H inductor. What is the net inductance of the combination?



$$.05 + .20 = 0.25\text{ H}$$



b) (15 points) In the LR circuit shown, $V_0 = 6\text{ volts}$, $R = 1\Omega$ and $L = 0.5\text{ H}$. \textcircled{A} is an ideal ammeter and \textcircled{V} is an ideal voltmeter (meaning that they don't affect the circuit).

i. What are the readings of \textcircled{A} and \textcircled{V} at $t = 0$, when the switch is first closed? Explain your reasoning.

At $t = 0$, the reading of \textcircled{A} is zero, since the changing magnetic field of L induces a current which opposes the change.

The reading of \textcircled{V} is $V_0 = 6\text{ volts}$

since there is no voltage drop across the resistor when there is no current through it.

ii. What are the readings of \textcircled{A} and \textcircled{V} after a very long time?

After a long time, the magnetic field of L is established and we have a d.c. situation:

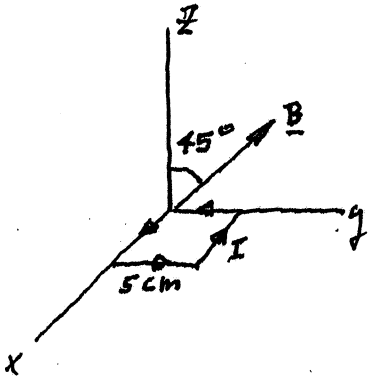
\textcircled{A} reads $\frac{V_0}{R} = 6\text{ A}$ and \textcircled{V} reads 0 .

iii. Roughly how long does it take for the readings to reach the final values in ii.

$$\text{time constant} = \frac{L}{R} = \frac{0.5}{1} = 0.5\text{ sec.}$$

c) (15 points).

i. A wire formed into a square, 5 cm on a side, is carrying a current, $I=3A$. What is the magnitude of its magnetic dipole moment, μ ? In which direction does it point?



$$\mu = IA = 3(.05)^2 = 75 \times 10^{-4} \\ = 7.5 \times 10^{-3} \text{ T-m}^2$$

μ points along the z axis:

$$\underline{\mu} = 7.5 \times 10^{-3} \underline{k} \quad \text{T-m}^2$$

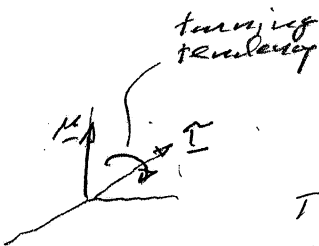
ii. A uniform magnetic field \underline{B} of magnitude $5 \times 10^{-3} \text{ T}$ and lying in planes parallel to the yz plane, while making an angle of 45 degrees with respect to the z axis, (as shown) is now turned on. What torque does \underline{B} exert on the wire loop? How will the loop move?

$$\underline{B} = \frac{|\underline{B}|}{\sqrt{2}} (\underline{j} + \underline{k})$$

$$\underline{\tau} = \underline{\mu} \times \underline{B} = \frac{\mu B}{\sqrt{2}} (\underline{k} \times (\underline{j} + \underline{k})) \\ = \frac{\mu B}{\sqrt{2}} (\underline{k} \times \underline{j}) = -\frac{\mu B}{\sqrt{2}} \underline{i} \\ = -\frac{(7.5 \times 10^{-3})(5 \times 10^{-3})}{\sqrt{2}} \underline{i}$$

$$= -26.5 \times 10^{-6} \underline{i}$$

$$\underline{\tau} = -2.65 \times 10^{-5} \underline{i} \quad \text{N-m}$$



The $\underline{\tau}$ acts to turn $\underline{\mu}$ into \underline{B}