Review before final exam

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Guide how to identify type of the problem

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The question is about?

acceleration (linear or angular) | force | conditions for system at rest

Only if the problem explicitly says "average acceleration" or if the acceleration is constant $\mathbf{a}=D\mathbf{v}/D\mathbf{t}$ may be used

Conditions for system at rest:

$a_x=0$

$a_y=0$

$\alpha=0$

The problem is for application of Newton's 2nd Law:

Does/can center-of-mass of any object move?

$m\ a_x=\Sigma_i\ F_{i\ x}$

$(0=)\ m\ a_y=\Sigma_i\ F_{i\ y}$

Usually a_y is zero for proper choice of coordinates

Circular motion?

$a_x=v^2/R$

for the x-axis pointing towards the circle center

Does/can any object rotate?

$I\ \alpha=\Sigma_i\ \tau_i$

Also often needed:

$\alpha=a/R$

$\tau=\pm r\ F \sin\theta$

or $\pm r_{-}\ F$

Rolling combines both for the same object

The question is about?

velocity

Only if the problem explicitly says “average velocity” or if the velocity is constant $\mathbf{v}=\mathrm{D}\mathbf{x}/\mathrm{D}\mathbf{t}$ may be used

(linear or angular)

Wave velocity?

$v = \omega/k = f\,\lambda$

A free fall problem?

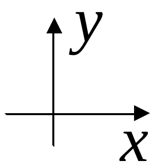
(the ***only*** force is weight)

$v_{fx} = v_{ix}$

$v_{fy} = v_{iy} - g\,\Delta t$

$\Delta x = v_{ix}\Delta t$

$\Delta y = v_{iy}\Delta t - {}^{1}/_{2}\, g\,(\Delta t)^2$



Some free fall problems are easier to solve using energy conservation

Collision?

(two objects, there is “before” and “after” the “interaction”)

Any rotation involved?

yes

Use conservation of angular momentum

$L_{tot\ i} = L_{tot\ f}$

Extended object: $L = I\omega$

Point-like object:

$L = \pm\, r\, mv \sin\theta$

or $\pm\, r_{-}\ \ mv$

no

Use conservation of linear momentum

$P_{tot\ i} = P_{tot\ f}$

$p = mv$

Does the text say “elastic” ?

yes

In addition, use $K_i = K_f$

no

Does the text say “perfectly” inelastic or the objects stick to each other ?

yes

$v_{1f} = v_{2f}$

Use conservation of mechanical energy

$E_{tot\ i} = E_{tot\ f}$

$K_i + U_i = K_f + U_f$

Extended object: $K = {}^{1}/_{2} I\omega^2$

Point-like object: $K = {}^{1}/_{2} mv^2$

Gravitational: $U = mgh$

Elastic (spring) : $U = {}^{1}/_{2} kx^2$

The question is about? position

(linear or angular)

A free fall problem?

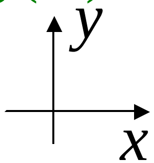
(the ***only*** force is weight)

$\Delta x = v_{ix}\Delta t$

$\Delta y = v_{iy}\Delta t - {}^{1}/_{2}\, g\, (\Delta t)^2$

$v_{fx} = v_{ix}$

$v_{fy} = v_{iy} - g\, \Delta t$



Some free fall problems are easier to solve using energy conservation

Use conservation of mechanical energy

$$E_{tot\ i} = E_{tot\ f}$$

$$K_i + U_i = K_f + U_f$$

Extended object: $K = {}^{1}/_{2} I\omega^2$

Point-like object: $K = {}^{1}/_{2} m v^2$

Gravitational: $U = mgh$

Elastic (spring) : $U = {}^{1}/_{2} k x^2$

Is velocity constant?

$$\Delta x = v\, \Delta t$$

Is acceleration constant?

$$\Delta x = v_i\, \Delta t + {}^{1}/_{2}\, a\, (\Delta t)^2$$

$$v_f = v_i + a\, \Delta t$$

linear	$\mapsto$	angular
x		θ
v	$\mapsto$	ω
a		α

Modification of the slide on "velocity" and "position" problems

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Is mechanical energy conserved?
(Is work by external or
non-conservative forces zero?)

yes → Use conservation of mechanical energy

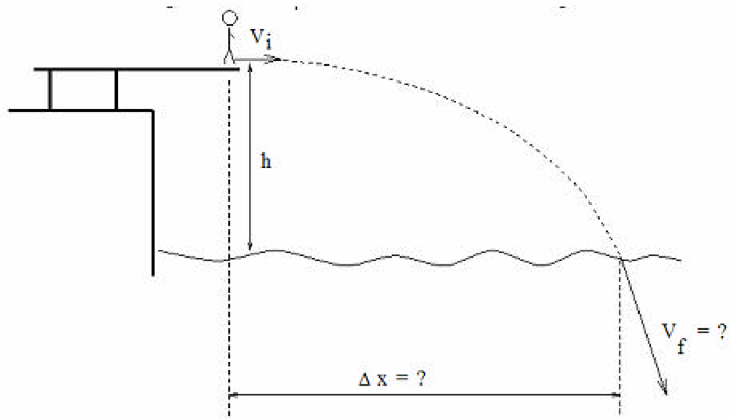
$$E_{tot\ i} = E_{tot\ f}$$

no → Use energy-work theorem

$$\Delta E_{tot} = W_{ext.\ or\ non\text{-}cons.}$$

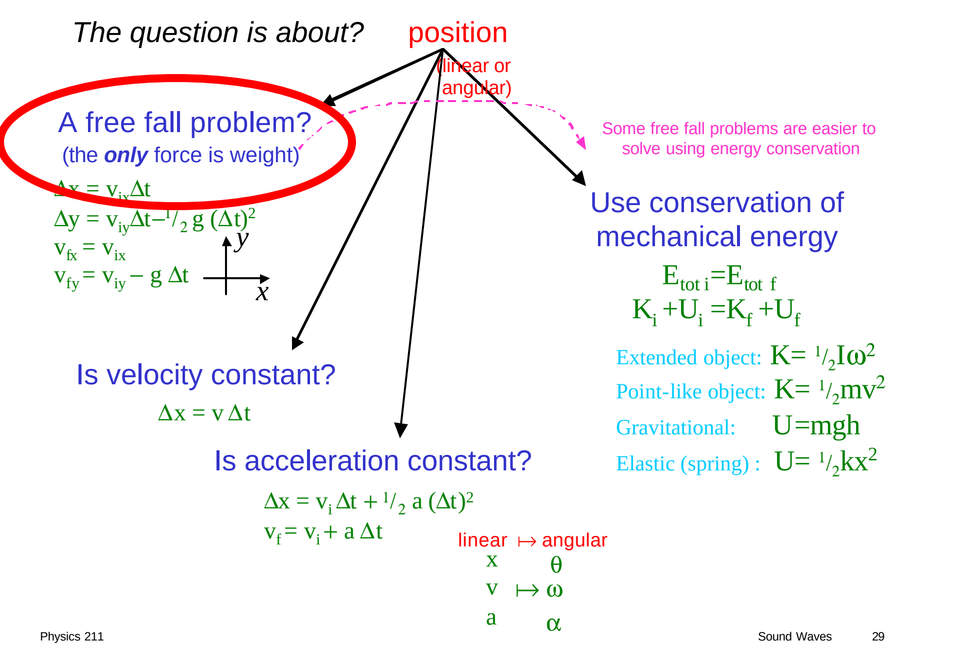
$$E_{tot\ f} - E_{tot\ i} = W_{ext.\ or\ non\text{-}cons.}$$

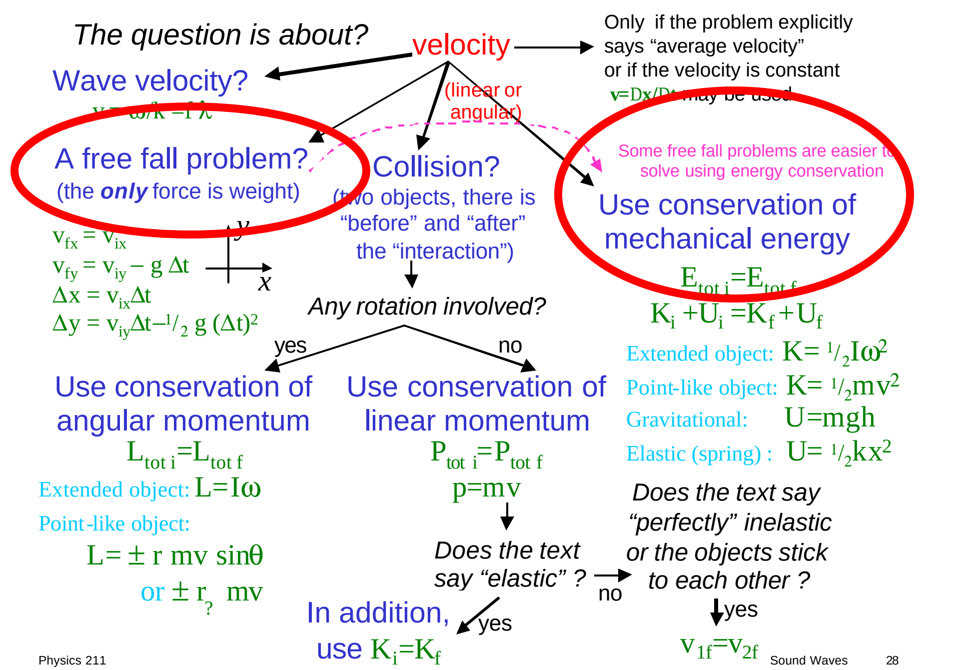
1. [15pts total] A diver runs off the diving board located at h=2m above the water with initial velocity v_i=3 m/s directed horizontally.



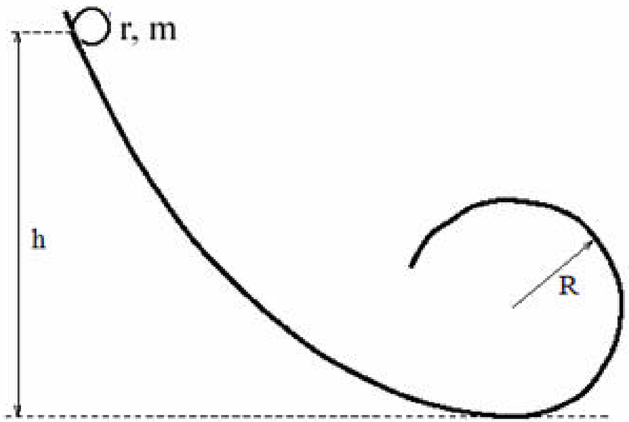
(1a) [10pts] How far does she fly in horizontal direction, Δx, before entering the water?

(1b) [5pts] What is her speed (i.e. magnitude of total velocity) v_f when she enters the water?



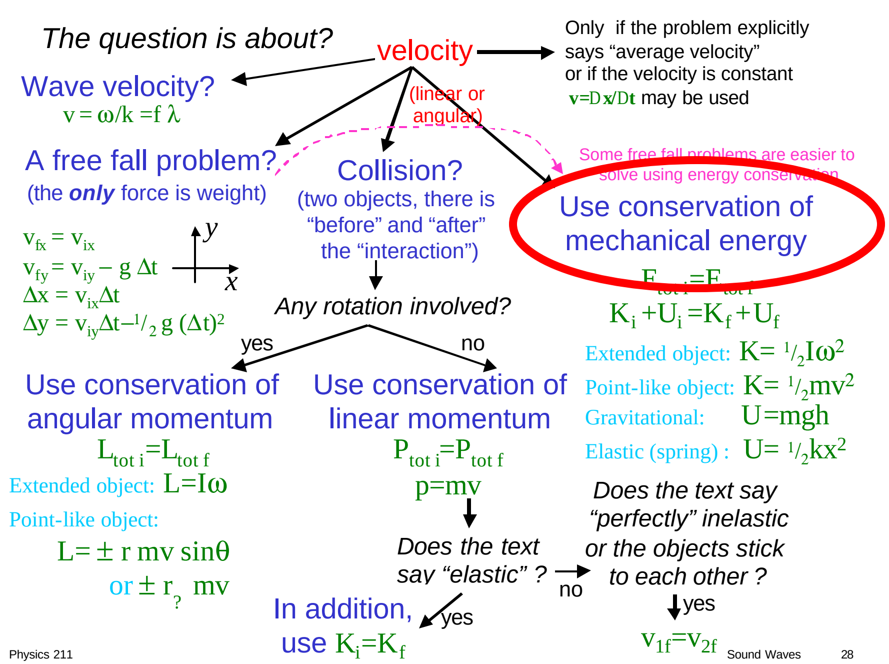


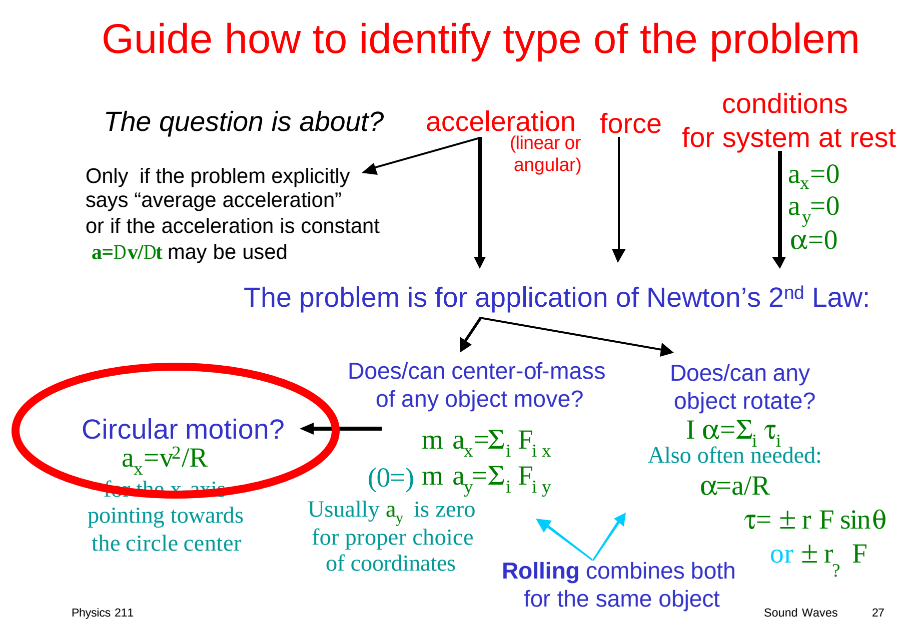
2. A uniform ball of mass r =0.1 m and mass m=3 kg rolls down without slipping along loop-the-loop track shown below. The radius of the loop is R=1.6 m. The ball is released from rest with its center at the height h = 12 m above the bottom of the track.



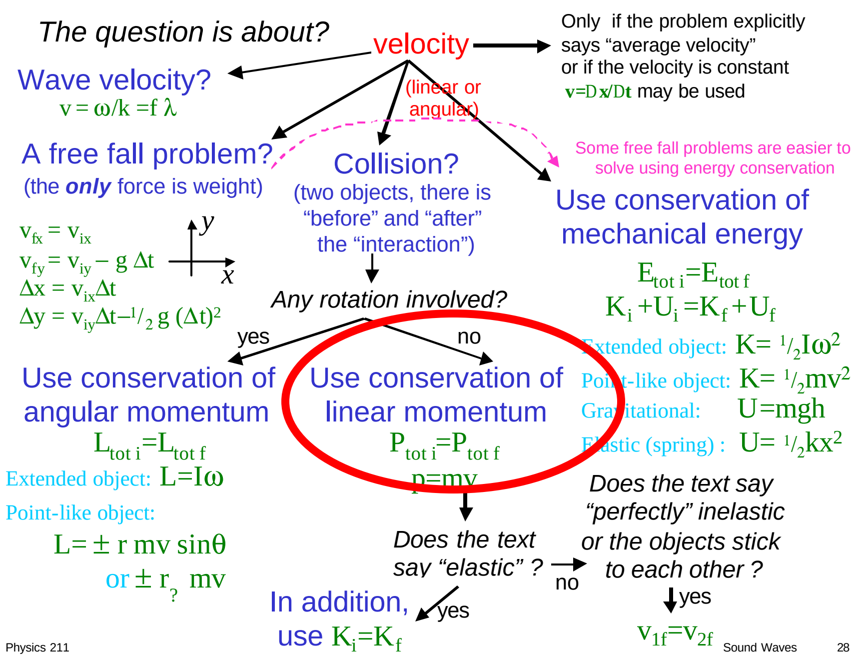
2b. What is the magnitude of the normal force exerted by the track on the ball at the top of the loop? ($g=10 m/s^2$)

2a. What is the speed of center-of-mass of the ball at the top of the loop? ($g=10 m/s^2$)

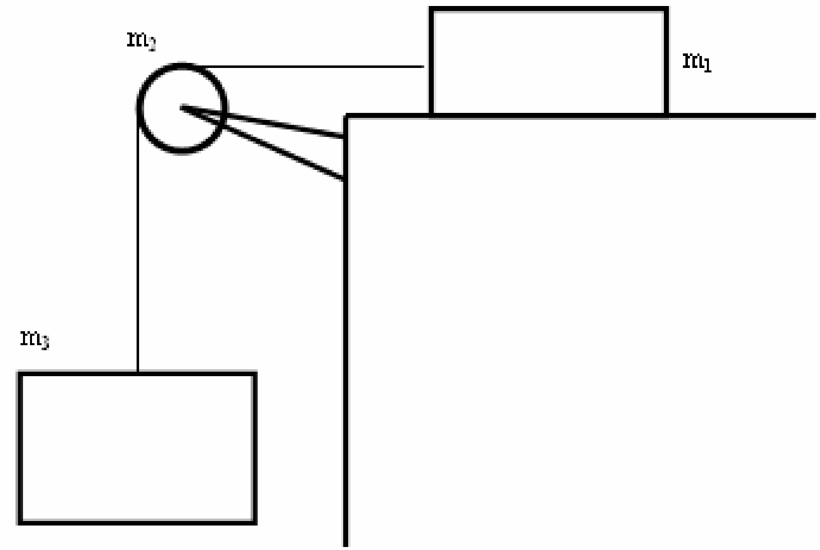




3. [10pts] A bullet is shot through a wooden block. The bullet has a mass of 0.003kg and its initial speed is 400 m/s. The block is initially at rest and has a mass of 5kg. The block has a speed of 5 m/s right after the bullet went through. Calculate the speed of the bullet after it went through the block.



4. Two blocks with masses m_1=7 kg and m_3=5 kg are connected by a massless string via pulley with a mass of m_2=6 kg. Assume the pulley is a uniform disk and that it rotates without a friction on its axle. The string is non-stretchable and doesn't slip on the pulley. Coefficient of kinetic friction for the block on the horizontal surface is μ=0.06. Find acceleration of this block assuming it is moving to the left (use g=10 m/s^2).



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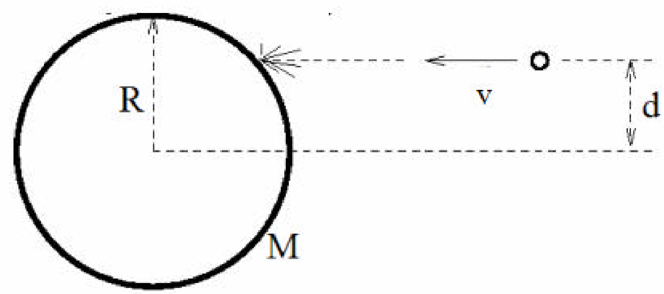
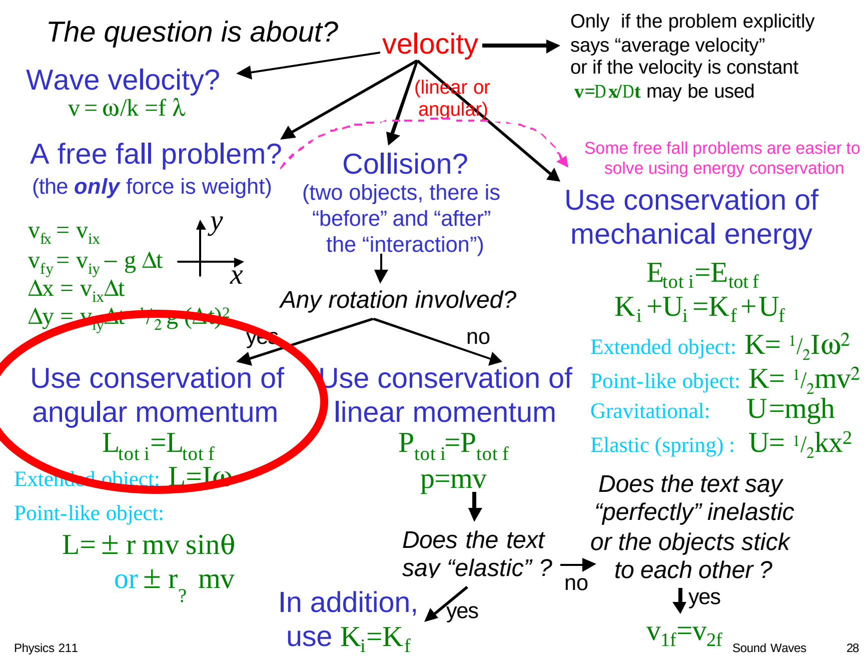
5. [10pts] Phobes is a small moon of Mars. It has a mass of M=5.8 10^{15} kg and a radius of R=7.5 10^3 m. For the purpose of the following problem, assume that Phobes has the shape of a uniform sphere and that it is initially at rest. Suppose a meteorite strikes Phobes at distance d=5 10^3 m off center and embeds itself inside Phobes, close to its surface. If the meteorite mass was m=3 10^8 kg and its speed was v=10^5 m/s, what is the angular velocity ω of Phobes about its axis of rotation after the collision?





6. [10pts] A block of mass m_1=3kg is suspended from the end of uniform horizontal beam of length L=7m and mass m_2=5kg pinned to the wall at the other end (i.e. it is attached to the wall using a hinge). The beam is suspended on a cable attached to its end creating an angle of θ=35° with the beam (see below). What are the horizontal (R_x) and vertical (R_y) components of the reaction force exerted by the pin on the beam?