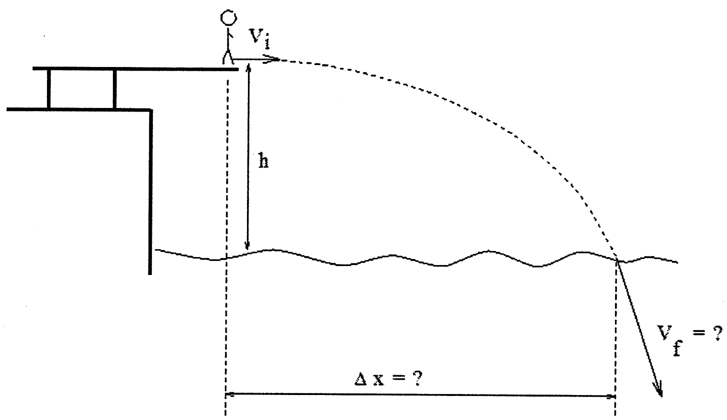
Practice problems before the Final Exam

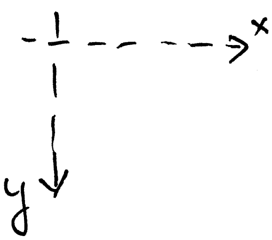
Problem Solving Activity **Name:** 2009

Here are practice problems taken from ***the final exams given in previous years.*** *This practice set does not cover all topics which are in the scope of the final exam. Therefore, it is strongly recommended to also review the past exams, the homework assignments and the workshop materials.*
Most of you will not be able to solve all these problems in one hour of workshop time. The solutions will be posted on the class web-site.

1. A diver runs off the diving board located at h=2m above the water with initial velocity v_i=3 m/s directed horizontally.



1a. How far does she fly in horizontal direction, Δx, before entering the water? (g=10m/s^2)



$$\begin{cases} \Delta x = v_i \Delta t \\ h = \Delta y = \frac{1}{2} g \Delta t^2 \end{cases}$$

$$\Delta t = \sqrt{\frac{2h}{g}}$$

$$\boxed{\Delta x = v_i \sqrt{\frac{2h}{g}}} = 3\sqrt{\frac{2.2}{10}} \approx 1.9\,m$$

1b. What is her speed (i.e. magnitude of total velocity) v_f when she enters the water? (g=10m/s^2)

$v_{fx} = v_i$

$v_{fy} = g\,\Delta t = g\sqrt{\frac{2h}{g}} = \sqrt{2hg}$

$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$

$\boxed{v_f = \sqrt{v_i^2 + 2hg}}$

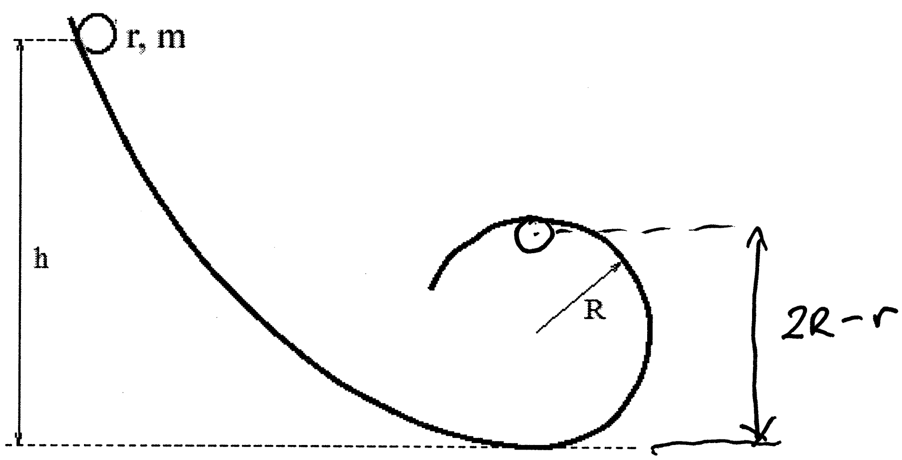
$= \sqrt{3^2 + 2 \cdot 2 \cdot 10} = 7\ \frac{m}{s}$

$E_i = E_f$

$\frac{1}{2} m v_i^2 + mgh = \frac{1}{2} m v_f^2$

$v_f = \sqrt{v_i^2 + 2hg}$

2. A uniform ball of mass r =0.1 m and mass m=3 kg rolls down without slipping along loop-the-loop track shown below. The radius of the loop is R=1.6 m. The ball is released from rest with its center at the height h = 12 m above the bottom of the track.



2a. What is the speed of center-of-mass of the ball at the top of the loop? (g=10m/s^2)

$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I_{cm} \omega^2 + mg(2R - r)$

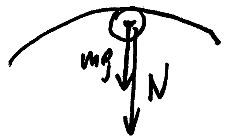
$\frac{1}{2} I_{cm}\omega^2 = \frac{1}{2}\left(\frac{2}{5} m r^2\right)\left(\frac{v}{r}\right)^2$

$gh = \frac{7}{10} v^2 + g(2R - r)$

$\boxed{v = \sqrt{\frac{10}{7} g (h - 2R + r)}} = \sqrt{\frac{10}{7} 10 (12 - 2 \cdot 1.6 \cdot 0.1)} = \sqrt{\frac{100}{7} 8.9}$

$= 11.3\ \frac{m}{s}$

2b. What is the magnitude of the normal force exerted by the track on the ball at the top of the loop? (g=10m/s^2)

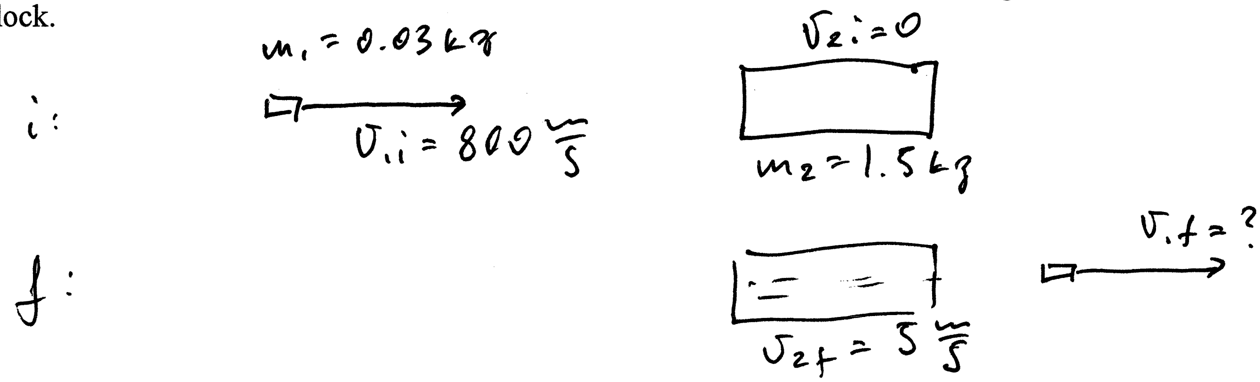


$$N + mg = m\frac{v^2}{R-r}$$

$$\boxed{N = m\left(\frac{v^2}{R-r} - g\right)}$$

$$= 224\ \text{N}$$

3. A bullet is shot through a wooden block. The bullet has a mass of 0.03kg and its initial speed is 800 m/s. The block is initially at rest and has a mass of 1.5kg. The block has a speed of 5 m/s right after the bullet went through. Calculate the speed of the bullet after it emerged from the block.



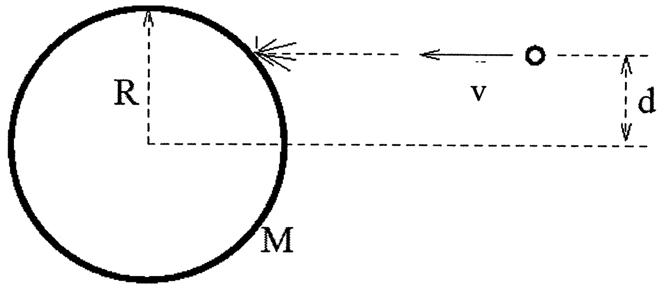
$$P_i = P_f$$

$$m_1 v_{1i} + m_2 \underbrace{v_{2i}}_{=0} = m_1 v_{1f} + m_2 v_{2f}$$

$$\boxed{v_{1f} = v_{1i} - \frac{m_2}{m_1} v_{2f}} = 800 - \frac{1.5}{0.03}5$$

$$= 550\ \frac{\text{m}}{\text{s}}$$

4. Phobes is a small moon of Mars. It has a mass of M=5.8 10^{15} kg and a radius of R=7.5 10^3 m. For the purpose of the following problem, assume that Phobes has the shape of a uniform sphere and that it is initially at rest. Suppose a meteorite strikes Phobes at distance d=5.0 10^3 m off center and embeds itself inside Phobes, close to its surface. If the meteorite mass was m=3.0 10^8 kg and its speed was v=1.5 10^5 m/s, what is the angular velocity ω of Phobes about its axis of rotation after the collision?



$$L_i = L_f$$

$$m v d = I \omega_f$$

$$I = \frac{2}{5} M R^2 + m R^2$$

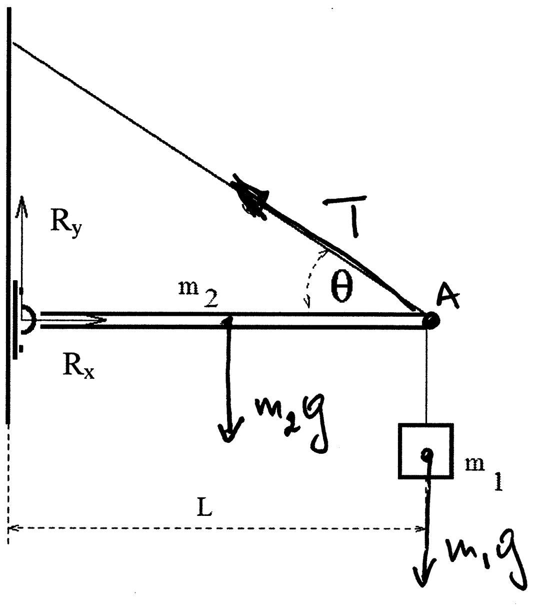
$$= 1.3 \cdot 10^{23} + \underbrace{1.7 \cdot 10^{16}}_{\approx 0}$$

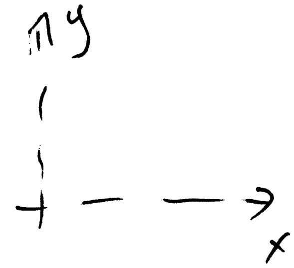
$$\boxed{\omega_f = \frac{m v d}{I}} = \frac{3 \cdot 10^8 \cdot 10^5 \cdot 5 \cdot 10^3}{1.3 \cdot 10^{23}}$$

$$= 1.7 \cdot 10^{-6} \ \frac{rad}{s}$$

$$= 8.4 \ \frac{\text{revolution}}{\text{year}}$$

5. A block of mass m_1=3kg is suspended from the end of uniform horizontal beam of length L=7m and mass m_2=5kg pinned to the wall at the other end (i.e. it is attached to the wall using a hinge). The beam is suspended on a cable attached to its end creating an angle of θ=35° with the beam (see below). What are the horizontal (R_x) and vertical (R_y) components of the reaction force exerted by the pin on the beam? (g=10m/s^2)





$$\sum_i F_{ix} = 0 \qquad R_x - T\cos\theta = 0$$

$$\sum_i F_{iy} = 0 \qquad R_y + T\sin\theta - m_1 g - m_2 g = 0$$

$$\sum_i \tau_i = 0 \qquad -R_y L + m_2 g \frac{L}{2} = 0$$

with respect to point A

$$\boxed{R_y = \frac{1}{2} m_2 g} = \frac{1}{2}\cdot 5 \cdot 10 = 25\text{ N}$$

$$R_x = T\cos\theta$$

$$T\sin\theta = (m_1 + m_2)g - R_y = \left(m_1 + \frac{m_2}{2}\right)g$$

$$\boxed{R_x = \left(m_1 + \frac{m_2}{2}\right) g \frac{\cos\theta}{\sin\theta}} = \left(3 + \frac{5}{2}\right)\cdot 10 \frac{\cos 35^\circ}{\sin 35^\circ} = 78.5\text{ N}$$

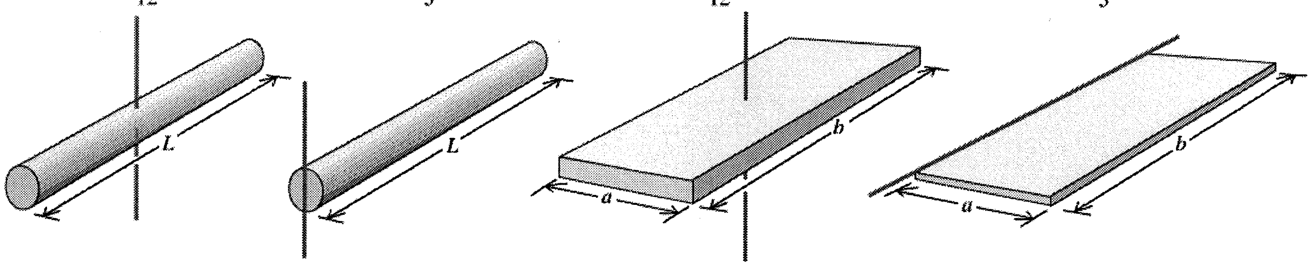
Table 9.2 Moments of Inertia of Various Bodies

(a) Slender rod, axis through center: $I = \frac{1}{12}ML^2$

(b) Slender rod, axis through one end: $I = \frac{1}{3}ML^2$

(c) Rectangular plate, axis through center: $I = \frac{1}{12}M(a^2 + b^2)$

(d) Thin rectangular plate, axis along edge: $I = \frac{1}{3}Ma^2$



(e) Hollow cylinder: $I = \frac{1}{2}M(R_1^2 + R_2^2)$

(f) Solid cylinder: $I = \frac{1}{2}MR^2$

(g) Thin-walled hollow cylinder: $I = MR^2$

(h) Solid sphere: $I = \frac{2}{5}MR^2$

(i) Thin-walled hollow sphere: $I = \frac{2}{3}MR^2$